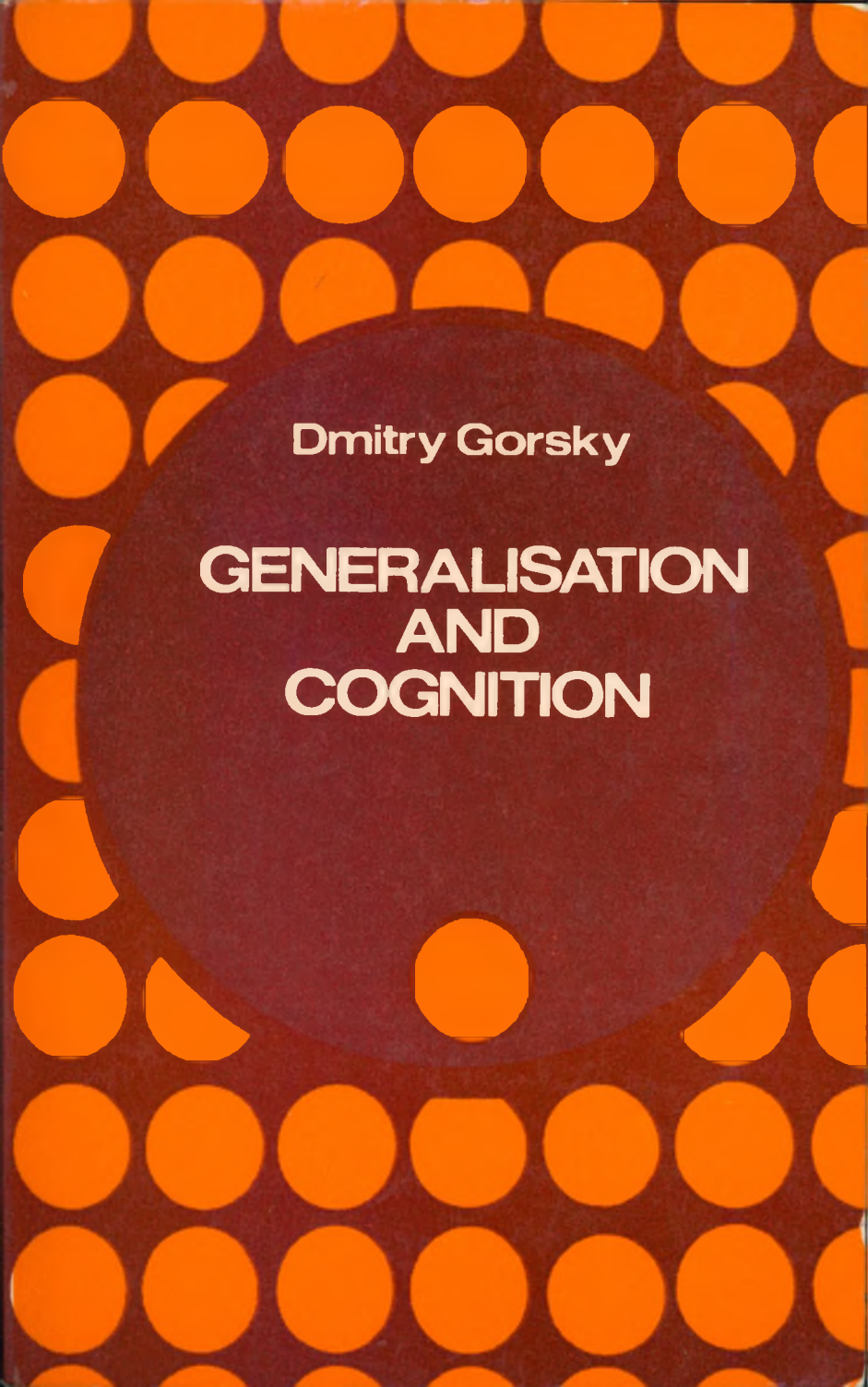




Dmitry Gorsky

**GENERALISATION
AND
COGNITION**

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GENERALISATION AND COGNITION

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ОБОБЩЕНИЕ И ПОЗНАНИЕ

На английском языке

The present book is the only one in the history of logic and the methodology of science to deal specially with the problems of generalisation.

It considers the forms of synthetic and analytic generalisation employed in scientific practice and also a broad spectrum of philosophical and methodological problems of generalisation that are of interest to the scientific intelligentsia.

Generalisation and Cognition also analyses the nature of individualisation and generalisation in fiction and poetry.

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FOREWORD

Generalisation is considered here on the logical and methodological level. That part of my book in which I try to apply modern logic to generalisation is a fragment of the logic of scientific knowledge.

The nature of the general, the relationship of the single and the general, the transition from the former to the latter, and from the less general to the more general in knowledge, and equally the interconnection of the individual in reality with the general in knowledge, and the application of the general to the particular have interested philosophical thought since its birth. These problems and analogous ones were already discussed by Aristotle in connection with his critique of Plato's doctrine about ideas, and in the doctrine about primary and secondary essence, etc. Lenin repeatedly drew attention to the importance and complexity of the problem of the dialectics of the particular and the universal, noting even Aristotle's 'helpless confusion' on this.¹

Generalisation plays an important role in transition from the individual and the particular to the general or universal, and from the less general to the more general. Many of the problems of the universal and the particular gave rise to the struggle of materialism and idealism, and of nominalism and realism in philosophy, a struggle that still continues.

The various extreme trends in the development of the sciences and their movement toward integration and differentiation (which have been revealed specially clearly in the age of the industrial and scientific revolution) were linked respectively with the processes of generalisation, broadening of concepts and theories, and their extrapolation on the one hand and with

their limitation, concretisation, and localisation on the other.

Scientific concepts, laws, and theories, which are in fact a generalisation of the particular (objects, facts, situations), irrespective of whether it has a scientific burden, are formed during scientific cognition. Not only is experimental knowledge (natural scientific and humanitarian) formed in that way, but also the analytical knowledge characteristic of mathematics, since it is only relatively analytical; the basic concepts of mathematics (given a historical approach to their origin) prove to have been formed from the generalisation of experience.

Given the specific character of the main spheres of knowledge (mathematics, natural science, and the humanities), a certain class of initial fundamental concepts of science is formed by one and the same means, the logical basis of which is constituted by 'definitions through abstraction'. The concepts and scientific theories so formed are then altered and developed during the historical, dialectical movement of knowledge. Generalisation plays an essential role in that process. Its various forms employed in empirical knowledge present interest for natural scientists.

My book examines the special features of generalising activity in the social sciences, in particular that connected with abstractions of higher order and with the idealising assumptions widely employed by Karl Marx in *Capital*. When abstractions are not absurd and not divorced from life but are a means of reflecting the essential in reality, they serve knowledge of practice incomparably better than a simple statement of the facts, and 'reflect nature more deeply, truly, and *completely*.'²

The concept of 'type', in contrast to 'class', is also introduced from L.A. Zadeh's theory of fuzzy sets, which has played an immense role in social science. Generalisations made in the humanitarian sciences on the basis of statistical studies are treated specially. Acquaintance with these social science aspects of generalisation is essential for the specialists in the humanitarian sciences, e.g. philosophers working in the fields of historical materialism and of ethics and aesthetics, political scientists, linguists, sociologists, etc.

The problematic of generalisation discussed here is treated in its epistemological, historical, and purely logical aspects, which is important for psychologists, who also study the process of generalisation on the plane of the individual's assimilation

of social experience. It is impossible to bring out the nature of generalisation at the level of teaching and education without reference to its philosophical and methodological aspects.

I also specially make a comparative study of the processes of generalisation in science and art (mainly on the example of fiction), which can interest specialists in aesthetics and the creative intelligentsia. Attention is paid to the difficulties of applying theories of a very generalised and idealised character in practice. Theories with idealisations cannot be applied in practice in a stereotyped fashion but only as a creative process. This problematic relates to the comprehension and concretisation of certain aspects of the Marxist-Leninist theory of practice, of its relation with theoretical knowledge, and of the practical application of scientific theories.

For people occupied with logic the analysis of the matter of the purely analytical generalisations to which we constantly resort in thinking may present interest.

The book contains a critique of neopositivism on the problems of generalisation (in particular on the question of so-called relevant generalisations), and demonstrates the illegitimacy of trying to make an absolute of logic.

I thus address myself to readers with a wide range of interests.

¹ See Lenin's conspectus of Aristotle's *Metaphysics* in his *Philosophical Notebooks* (V.I. Lenin. *Collected Works*, Vol. 38, Progress Publishers, Moscow, 1980, p. 366).

² See Lenin's conspectus of Hegel's *The Science of Logic*, *op. cit.*, p. 171.

INTRODUCTION

When we formulate the analytical definition of a certain term on the logico-methodological level, we strive to get its meaning to embrace the cases in which it has been employed in various sciences and in various contexts of specific scientific languages before it was defined. The development of the analytic definition of a term is usually associated, however, with a certain refining and concretising of its sense. That brings out the synthetic properties of the definition. At first we cite the traditional analytic definition of the generalisation, and then extend and concretise it, introducing characteristics of synthetic generalisation.

By *generalisation* is understood, as a rule, the transition from knowledge of the particular to knowledge of the general, from knowledge of the less general to that of the more general, and also the results of these transitions recorded in corresponding concepts and judgements. That understanding of generalisation holds in most definitions of the concept of analytic modes of generalisation based on experience of use of the term 'generalisation' in the most varied contexts.

As an example of this definition of generalisation let us take that proposed by Polya:

Generalization is passing from the consideration of one object to the consideration of a set containing that object; or passing from the consideration of a restricted set to that of a more comprehensive set containing the restricted one.¹

In this definition the concept is explained extensionally, i.e. through the range of the ideas and through extensions

of terms that are in a relation of generalisation to one another.

A broader definition of generalisation would embrace both its analytic and its synthetic forms. It may be formulated as follows. By *generalisation* we understand transitions: (a) from the idea of the individual included in a concept, judgment, norm, hypothesis, question, etc., to the corresponding idea of the general; from ideas of the general to ideas of the more general (an extension of the range of the idea, and sometimes of only some of its components, is characteristic of these transitions, but in the case of generalisation of theories with a limiting transition concretisation of the theory's domain is characteristic); (b) from separate facts, situations, events, objects, and phenomena to their identification in ideas, and the formation of general concepts and judgments about them (including statements with the form 'most *S* are *P*'). This process of identification is a *sine qua non* of the formation of corresponding hypotheses, theories, and conceptions. The concepts and judgments obtained through these transitions are often called generalisations (having in mind the method of obtaining them); the resulting generalisations differ in content from the initial objects of generalisation.

I shall mainly make specification of the different types of generalisation, analytically, through study of various contexts (scientific and natural language), relying on the general definition of generalisation given above.

All types of generalisation can be divided into analytic,² and synthetic ones.

By *analytic* I shall mean generalisations made without direct reference to experience, on the basis of an understanding of linguistic expressions, of definitions not based directly on experience, and of application of the rules of deduction to them. *Synthetic* generalisations are made through analysis of experience, including application of inductive procedures to it, procedures that are often called the rules of generalisation.

Examples of analytic (sometimes called formal logical) generalisations are the mental transitions from the concept 'right-angled triangle' to the concept 'triangle', from the concept 'mechanical form of the motion of matter' to the concept 'form of the motion of matter', from the statement 'lead is an elec-

tric conductor' to the hypothetical statement 'all metals are electric conductors', from the question 'is the problem resolvable in the given case (with the given value of the parameters)?' to the question 'is the given problem resolvable in the general case?', from the juridical norm 'theft is forbidden' to the norm 'embezzlement is forbidden'. The mental transition from the hypothesis 'there is life on Mars' to the hypothesis 'there is life on all the planets' is an example of analytic generalisation of a hypothesis.

Examples of synthetic generalisation are the modes of forming concepts created by means of identification abstraction, on the basis of 'definitions through abstraction' and also generalisations of scientific theories through concretisation of the subject domain and imposition of certain limitations on it, and so on.

The process of synthetic generalisation is associated with the forming and development not only of concepts and scientific theories but also of statements.

During the forming of concepts they are often generalised into concepts through identification according to certain attributes (for example, by comparison of the objects studied with a certain standard). The new concepts for science, and for knowledge in general, formed in this way, I shall call *primary, elementary generalisations*.

Let us assume then that a certain primary generalisation, and a set corresponding to it, have been formed in the course of cognition. The point may arise of whether a certain property is inherent or not in all the elements of the set. When that has been resolved by inductive analysis of this set, certain simple statements can be formulated. If they prove to be generally affirmative or generally negative, they can be called *primary inductive generalisations*. These can be formulated as well through generalisation of corresponding single statements. Several general dependences are thus formulated from study of the tables in which the results of the single experiments are recorded.

Sometimes, instead of this division of generalisations into analytic and synthetic a rather different and more detailed division is accepted. Thus, taking into account Lazarev's and Novoselov's points about the classification of types of generalisation

they can all be divided at a certain level into analytic, synthetic, and extensions. The last, for example, include the generalisation of scientific theories with a limiting transition.³

For the sake of popular presentation, I shall first consider the modes of synthetic generalisation (Chapter I), and then pass to analytic methods of generalising activity (Chapter II). In sections 5 and 6 of Chapter II I shall discuss problems associated with both analytic and synthetic generalisations.

It is also essential to divide all types of generalisation into *generalisations on the synchronic plane* (logical and structural) and those *on the diachronic plane* (genetic, inductive-historical).

The classification of types of generalisation into *creative* and *non-creative* presents interest. It is based on comparison of the initial and the generalised knowledge. If the generalised knowledge differs from the initial in level of abstraction, degree of idealisation, level of explanation and interpretation, and change of the subject domain, we are dealing with creative generalisation, in the opposite case with non-creative generalisation.⁴

The reverse process of passing from the more general to the less general and particular is called *limitation (determination)*. Concepts, statements, norms, questions, etc., and also theories and conceptions, can be delimited (restricted).

The analytic processes of limiting general knowledge need to be distinguished from those of applying general knowledge in practice, viz., to separate objects and situations in the world around us. In this case we shall be dealing with so-called *localisation* (see Chap. II, § 1 and Chap. III, § 4).

Generalisation is closely associated in the thought process not only with processes of limitation but also with those of the definition and division of a concept, and with methods of abstraction and idealisation. During my inquiry I shall turn to explanation of the relations between the process of generalisation and the other mental techniques mentioned above.

³ George Polya. *How to Solve It*, (Princeton U.P., Princeton, N.J., 1946,) pp 97-98.

- ² For the different meanings of the term 'analytic', see Chapter II, § 6.
- ³ See: F.V. Lazarev and M.M. Novoselov. The methodological significance of the problem of precision in the development of scientific theories. In: L.S. Iliechev (Ed.), *Materialisticheskaya dialektika kak obshchaya teoriya razvitiya* (Materialist Dialectics as a General Theory of Development), Book 2, Nauka, Moscow, 1982, pp 250-280.
- ⁴ Lazarev and Novoselov divide generalisations into ones generating new semantic units and those that do not (see the entry 'Generalisation' in the *Great Soviet Encyclopaedia*, Vol. 18, Macmillan Inc., New York, p 107).

Chapter I

ABOUT SYNTHETIC MODES OF GENERALISATION

§ 1. Identification abstraction and the process of generalising objects in concepts, and facts in laws

The sense of *identification abstraction* consists in certain objects in the domain regarded being united in a class (set) on the basis of their common properties, in accordance with which they are distinguished from other objects of the said domain. During the identification abstraction there is an abstraction from the properties that individualise the separate objects united in the class, and also from the properties that are inherent in each of the elements of all their regular, non-empty subclasses.

During identification abstraction concepts are formed that are characterised by the existence on the one hand of a content (of an intensional, i.e. an aggregate of the general properties specific to them), and on the other hand of a range (extensional) that is a set of objects thought of as indiscernible in regard to the properties fixed in the content of the concept.

Since the common is singled out in objects when they are being identified, and the objects are identified in a class from the standpoint of this common feature, i.e. any one of them can therefore represent all the other elements of the class ('the whole class'), identification abstraction is often regarded in the history of philosophy and logic as *generalising*, as generalisation of the objects studied by identifying them.

This process of the formation of classes of objects identical with one another is essentially based both on the process of identification, i.e. of their generalisation by some property, and on *indiscernibility abstraction*. Formation of identical objects presupposes indiscernibility of the generalised objects by the properties by which they may differ from one another. Indiscernibility abstraction can be treated as a stage in the processes of

generalising and identifying the objects studied into abstract object-classes. Indiscernibility abstraction is related to identification abstraction as a part to the whole.

The process of identification abstraction presupposes, however, that we can somehow distinguish the elements of the classes of objects formed by generalisation from one another (for example by their names). Otherwise all the elements considered from the angle of their general and specific properties would be merged into one element.¹ But to differentiate the elements of a class by their names it is necessary first to differentiate them by specific characteristics. Consequently, something like a vicious circle arises.

A certain epistemological difficulty thus arises when identification abstraction is being analysed. In the course of it we always disregard the individual characteristics of the objects studied. If such disregard meant 'absolute neglecting' of these properties, the elements of the sets so formed could not be distinguished from one another by name since such a distinction is also associated with differentiation by their properties.

This disregard does not signify 'absolute neglecting' of the distinctive properties of the particular and remembering only the general and specific properties of the generalised objects. During abstraction, we continue to bear in mind, while disregarding the concrete individual properties of the separate objects, that each of the generalised objects can have different individual properties, and can be individualised. In other words, the processes of identification abstraction and of generalising objects in a concept cannot be adequately described without bringing in the category of possibility.

The individual properties of separate objects and the general properties of regular subsets of a set are often abstracted in such a way that we retain as a possibility the means of reproducing them subsequently in thought, which also means in the language of logic that they take on the character of variables that comply with the meanings from which they have been abstracted.²

When the properties of the objects that we are abstracting from during the forming of a general concept of them have the form of alternative characteristics for each of the objects, it may mean that any object has an additional alternative (though

it is not known which) in the concrete case in the conditions of this abstracting. Thus, when we develop a general concept of mechanical movement we abstract from its rectilinearity or curvilinearity, its uniformity or non-uniformity, etc. We then mean, however, that in each concrete case there is one of the indicated alternative characteristics of motion (but it is not known which).

In ordinary language there are not only single terms for defining separate objects, but also general ones for defining classes of objects that generalise certain single objects. This means that each separate object is included in the system of connections of the general, while the general is always potentially correlated with the particular. This dialectical thought is clearly expressed by the character Satine in Maxim Gorky's play *The Lower Depths*: 'What is man? ... It's not you, not me, not them—no! It's you, I, them, the old 'un, Napoleon, Mahomet ... in one!'³

A dual kind of problem can be resolved in the course of identification abstraction (or generalisation): (1) the objects are presented and the task is to identify and generalise them; and (2) the properties are indicated and it is required to select only those objects from the studied field that possess them. The properties by which the studied objects are identified are sometimes known as the *identification abstraction interval*.⁴ They may also be called *mutatis mutandis* (with appropriate changes) the *generalisation interval*. The essence of the procedure of identification abstraction thus boils down to the forming of appropriate abstract objects or sets in which all the elements are 'stuck together' as it were, by some property common to them with the result that any concrete element can represent the whole set and correspondingly the property by which this set is defined. Asserting that the classes of abstraction and their specifying properties can be represented by their elements is the essence of the *principle of abstraction*, which is expressed, consequently, in the dialectical manifestation of the general through the particular, and in their interconnection.

We encounter the principle of abstraction not only when a set (class) is formed through selection of objects possessing a certain property P (or rather, objects satisfying the propositional function x is P or $P(x)$), in the course of which we

convert them into elements of a class 'stuck together' or identified according to property P), but also when they are formed through subdivision of the domain D into sets without common elements by way of a relation of the equality type (see Chap. I, § 3).

The generalisation of facts in concepts is thus a problem (a) of identity, of identifying the studied objects in one object, when they become indiscernible in a certain sense, and (b) of identifying the object with itself.

Ontological and epistemological aspects can be distinguished in the problem of identity: Here I am interested in the epistemological aspect, viz., the creation of a model of identity as a reflection of identity in reality, and definition of identity and its application. Gorskaya distinguishes four types of identity on the synchronic plane: purely formal (Leibnizian), quasi-concrete, concrete, and identification on the basis of definitions by abstraction.⁵

In Leibniz' definition of identity conditions are established in which the given object may be regarded as the same object. It is formulated as ' $x = y$ ' when, and only when, all the properties belonging to x also belong to y , and vice versa. It can be written symbolically as follows:

$$(x = y) \equiv \forall P (P(x) \leftrightarrow P(y)).$$

This *formal* definition has several drawbacks in the opinion of specialists; it is unconstructive since it is necessary, in order to establish the identity of the objects x and y (i.e. the fact that they represent one and the same object), to review all the properties P , the number of which is unlimited in real objects. Furthermore the determinant is hardly any clearer intuitively than the determined property; the concept of property (or rather 'of all the properties of an object') is hardly any clearer in the intuitive sense than the concept of the identity (equality) of the objects. Such a definition can only be employed to identify abstract mathematical objects considered in a certain system of knowledge, since such objects are assigned to a system on the basis of a limited number of inherent properties.

In order to identify a real object with itself Gorskaya suggests employing *limited, concrete identity* ('quasiconcrete syn-

chronous identity'). This presupposes a review of only the essential properties rather than of all of them. Uemov considers that

Man identifies things in his practical activity without making an attempt to establish the generality of all their properties separately. So we recognise a friend when we see him without taking his temperature, measuring his blood pressure, counting his pulse, measuring the oval of his face, and so on.⁶

When we identify *different* objects, i.e. in the course of generalising the difference by certain common properties, we employ *concrete identity*. In that case we do not apply Leibniz's law in its unlimited formulation; when generalising different objects we single out certain properties common to them, and disregard the difference between the studied objects, and between the regular subsets of the given sets. Thus, when generalising animals by the property 'of possessing a compound stomach' (i.e. forming the concept 'ruminants'), we disregard not only the differences between separate cows, separate horses, etc., but also the differences between elements inherent in the set of cows, the set of horses, the set of camels, the set of dogs, and the sets of domestic animals (sheep, pigs, goats, cats, etc.).

The last type of the identification and generalisation of objects is called *definition by abstraction* (I shall analyse this method specially).

Gorskaya, characterising the types of identity listed above, writes:

Only the first of them is extremely abstract and formal, therefore it is not employed in the Leibnizian formulation (when analysing real material objects—*D.G.*). In all other types of identity we meet elements of dialectics: the particular and the universal, concrete and abstract, finite and infinite, essential and inessential, etc. It is therefore also possible to employ them when analysing scientific problems.⁷

A stage must be distinguished in the general course of a concept's formation and shaping, and also further stages connected with its perfecting, alteration, and development. I shall examine the process of the forming of a concept and its subsequent stages on the plane of the historical genesis of knowledge. In that case the individual's assimilation of accumulated social knowledge will be of secondary importance for me.

The results of the formation of a concept on the historical plane, which are not subjected to further perfecting, alteration, or development, I shall call *primary or initial generalisation*, and the classes corresponding to primary generalisations primary or initial classes of generalisation.

The point about the formation of primary classes of generalisation (the formation of concepts associated with the distinguishing of certain general fundamental properties in the objects studied) is epistemologically the most difficult and complicated of the questions arising in connection with investigation of the process of synthetic generalisation. What do some of these difficulties consist in?

A set is usually considered defined in a certain strict, constructive sense, if it can be said of each object investigated on the basis of quite simple rules that it is an element of this set. The set of members of a national philosophical society can be considered defined in the strict sense, for example, by the attribute of being included in the lists of the society (it being assumed that these lists exist and have been correctly compiled). The sets of even numbers, the continents of the globe, and planets of the Solar system are defined in the strict sense. But most sets of objects relating to nature and the social sphere of reality are not defined in the strict sense. Such are the sets of fish, metals, animals, plants, handsome people, noble deeds, good novels, reminiscences. The sets of animals and plants are not defined in the strict sense since there are whole types of living creatures about which there is no unanimity among scientists about whether they are animals or plants. It is quite difficult to compile a set (class) of noble deeds, handsome people, or good novels, even when it is done by quite competent people.⁸ We run across difficulties at every step when drawing hard and fast lines (as Engels pointed out). The set of every person's reminiscences is extremely indeterminate and depends on his mental state, and the time and the place where the set is compiled (see Chapter III, § 5, about fuzzy sets). All that is evidence of the complex dialectical character of the reality around us.

But the needs of scientific research and social practice compel us to define sets, if not in the strict sense mentioned above, at least in order to make them correspond to the ever-

growing requirements of constructiveness. The point is that in order to form primary classes of generalisations, and to define them in a sufficiently strict constructive sense, we resort to a *constructivisation of reality*, by which is meant the mental process that ensures the establishing of quite 'hard and fast' boundaries between separate objects of the surrounding world and their classes. This is connected with the development of quite simple, easily applied criteria and rules that make it possible to say that some object of reality is an element of the set that interests us.

The crux of this activity can be demonstrated from the example of the forming of the concept of a set of voters and the corresponding development of a certain primary class of generalisation. To take part in elections a person must satisfy certain conditions: be a citizen of the country, be mentally healthy, etc. Let us assume that the question of a voter's age qualification comes up. As a citizen taking part in elections one person may understandably mature socially earlier and another later. Analysis of that circumstance in essence and the taking of a corresponding decision as regards each voter's age qualification would be quite a complicated ticklish business fraught with debatable situations; furthermore, the analysis and decision would call for much energy, time, and money from society. Society therefore decides the matter of the age limit for voting more simply and constructively; it sets a certain constructivised limit, say 18.

This limit is formal in two senses. (1) It does not take into account that some young people deserve to be voters earlier than 18 years of age (many Soviet youths who had not reached 18 during the Civil War and the Great Patriotic War performed heroic deeds for the Motherland). Civic maturity may come later in others.

(2) This limit makes it possible to decide whether a person may vote or not simply and constructively, in a purely formal way, by an age criterion (with observance of the other conditions as well). At the same time it is also rational, since experience indicates that a person has become a responsible citizen by that age, as a rule, and may take part in elections.

In many cases the generalisation of an object and con-

struction of classes of primary generalisation (and likewise the formation of corresponding concepts) are done by application of certain general, simple rules to the initial, quite 'hard and fast', constructivised objects. In the individual case this may be the rule of searching out some common property P in the generalised objects by comparing them. Through application of this rule we form a concept $xP(x)$ about them (where x 's that have property P , for example, are those that possess the property 'of being a chemical element') and a corresponding set or class $WxP(x)$ (the set of those x 's that possess property P , for example, the set of x 's that are chemical elements).⁹ $P(x)$ will thus be the content of the concept $xP(x)$, and the class $WxP(x)$, corresponding to the concept $xP(x)$ will be its range. Similarly we can form a concept $xP(x)$ according to the property $P(x)$ and a class (range) $WxP(x)$ corresponding to it.

If we have derived primary generalisations of the objects studied according to different properties $P_1, P_2, P_3 \dots P_n$ and have formed corresponding concepts $xP_1(x), xP_2(x), xP_3(x) \dots xP_n(x)$, and also the classes $WxP_1(x), WxP_2(x), WxP_3(x) \dots WxP_n(x)$ appropriate to them (classes of primary generalisation), we can ask whether certain other general properties $Q_1, Q_2, Q_3 \dots Q_n$, are inherent in each element of the primary classes of generalisation we have formed, i.e. pose the question of an extension of the interval of abstraction (generalisation) and so of the content of the concept on the basis of subsequent generalisation. In other words, we can build hypotheses (predictions) about the inherency of properties Q_i in the class WxP_i .

The process described above (at least in its simplest variants) is usually realised in the course of real knowledge as *primary inductive generalisation* (investigation). Assume that we have formed a class of metals at the phenomenological level (i.e. a certain primary generalisation). Its symbolic expression will be: $WxS(x)$, where S is the property of being a metal.¹⁰

Now we can ask whether the properties of 'being an electric conductor', 'a heat conductor', etc., is inherent in each of the elements of the class $WxS(x)$. We thus assume that the properties of being an electric conductor or heat conductor

have not figured in the primary process of generalisation, i.e. during formation of the concept of metal ($xS(x)$) of the class ($WxS(x)$) corresponding to it. In other words we assume that if the property of being a metal is rather complicated, introduced as an abbreviation for the conjunction of properties $P_1^1, P_2^1, P_3^1, P_4^1$, etc., then the properties of being an electric conductor and being a heat conductor are absent in the conjunction of properties P_1^1 . This means that the problem of whether the properties of being an electric and a heat conductor are inherent in the class $WxS(x)$ (as I shall express it when it is a matter of the inherency of these properties in each element of class $WxS(x)$) is resolved by way of experimental, synthetic investigation, and not analytically.

Consider the simplest inductive deduction—inference by incomplete induction based on analysis of several statements:

- (1) 'Iron is an electric conductor'
- (2) 'Copper is an electric conductor'
- (3) 'Tin is an electric conductor'
- (4) 'Lead is an electric conductor'
- (5) 'Aluminium is an electric conductor'
- (6) 'Nickel is an electric conductor'
- (7) 'Iron, copper, tin, lead, aluminium, and nickel belong to the class of metals'

-
- (8) 'All metals are electric conductors'.

This deduction expresses a hypothesis that the property of electric conductivity (property Q) is inherent in all metals (class $WxS(x)$) on the basis of the results of experimental study of only some members of the class of metals. In this case, since the class of metals is finite, testing of the hypothesis (i.e. of a certain primary inductive generalisation) boils down to making an experimental investigation for representatives of metals that were outside the field of vision in the statements introduced above.

The inference generalises members of the class of metals by a property that is not specific to this class. That circumstance is the basis for including the class of metals in a broader class of conductant substances ('metals are conductant substances'), i.e. for a further generalisation of the class of metals. If conclusion (8) is true, then the property of being an

electric conductor can be joined conjunctively to the previous properties of the concept of metal, among which there are also specific ones. The generalisation interval of the elements of the class 'metal' thus grows.

We not only construct hypothetical statements through inductive inferences of this sort but also enrich the initial generalisations in content. The initial inductive generalisations, i.e. certain statements of even the simplest type, are thus a way of deepening the corresponding initial generalisation (concept).

When examining the even numbers 6, 8, 10, 12, etc., in a similar way, we note that each of them can be represented as the sum of two prime numbers: $6 = 3 + 3$; $8 = 3 + 5$; $10 = 3 + 7$, $12 = 5 + 7$, etc. From that we can draw the hypothetical conclusion that all even numbers can be represented as the sum of two prime numbers (Goldbach's proposition). If this proposition is proved the initial generalisation (concept) of an even number would be deepened and enriched. If propositions (1) — (6) of the inductive reasoning above are treated as proof of hypothesis (8),¹¹ and if the truth of proposition (8) is proved, it will be converted from a hypothetical generalisation into a categorical initial inductive generalisation.

In the example of inductive reasoning above the truth of the hypothesis is demonstrated comparatively simply by the hypothetical deductive method (Jevons' method): conclusions follow from the hypothesis about the electric conductivity of the other metals ('silver is an electric conductor', 'gold is an electric conductor', etc.), whose number is limited. The conclusions are verifiable experimentally. Since the result is positive, the property of being an electric conductor proves to be inherent in all metals, and the class of metals investigated is generalised by a new, common property.

The process of substantiating a hypothetical generalisation about the common properties of a finite class of studied objects (not to mention the grounds for hypothetical generalisations about practically infinite classes) is often quite complicated. Kostyuk cites the following example of the testing of a hypothetical generalisation connected with a conjecture about the origin of planets:

Laplace suggested in his day that the planets were formed

simultaneously with the Sun. Later this hypothesis was refuted on the grounds that the mass of the planets required by it was greater than their observed mass. Subsequently, when the chemical composition of the Sun and planets became better known, there proved to be a possibility of the planets' having lost a great deal of hydrogen and helium. Allowing for that loss the observations that were considered 30 or 40 years ago to oppose Laplace's hypothesis now confirm it. Other facts have also appeared that favour it. Such a situation is the rule rather than the exception in the development of scientific knowledge.¹²

It is important, when a hypothesis is being tested, to check both the consequences derived from it (including the trivial ones) and the results that are particularly rich in information and are heuristically new.

Initial generalisations are not necessarily linked with the generalisation of objects by their properties (single-member predicates correspond to them). They may also generalise relations (multi-member predicates correspond to them), above all two-member relations. In the process of cognition we often make the transition from relation R between a pair of separate objects a^* and b^* to a set of such pairs: $W(a, b) R(a, b)$, i.e. 'the set of pairs a and b that are connected by relation R '. The initial generalisation of the concept will have the form $(a, b) R(a, b)$, i.e. 'the objects a and b that are connected by relation R '.

The formulation of laws of science is connected with establishing interconnections and mutual relations between the characteristics of the objects studied, including the development of causal relations. Bacon-Mill methods play an essential role in the development and generalisation of generalisations of causal dependences, and their significance in modern science is great.

Depending on whether we are dealing with abstract, quantitative time or historical time,¹³ the laws of science may take a different form: (1) $\forall x (P(x) \supset Q(x))$ ('if x is a metal, then x is an electric conductor'); (2) if $P(x, t_1)$, then $Q(x, t_2)$, where t_1 and t_2 are parameters fixing different points of historical time, and $t_1 < t_2$ ('if the Caspian Sea evaporates at the same rate as at present (t_1), it will cease to exist in a certain time (t_2)').

In the context of the application of abstract-quantitative

time all laws can be divided into causal and immanent ones. Examples of causal laws are as follows: 'all metals expand when heated'; 'the volume of a gas inversely proportional to the pressure on it', and so on. Examples of immanent laws are: 'all metals are electric conductors'; 'in a system that is thermally isolated, entropy will increase during irreversible processes', and so on. They record the states of things in which certain characteristics are immanently inherent in objects and systems of objects; the existence of these characteristics is not due to the action of causes of any kind. Both types of law are generalisations of the facts of reality.

Initial generalisations may get a different interpretation in the course of cognition, be incorporated into scientific theories, be given a mathematical description, be tested by the hypothetical-deductive method, and so on. Analysis of these problems is outside the scope of this book.

Initial generalisations are made during cognition not only as regards objects that exist at one and the same time, i.e., in the context of abstract-quantitative time, but also as regards objects that are in the context of historical time. A generalisation can thus be made about the general concept of reflection at the level of the living, having in mind different stages of development of the forms of reflection of reality. The biological form, of course, includes reflection at the premental and mental stages. The social form, specific to man, includes prescientific and scientific knowledge.

Each of these stages may be individualised, and initial generalisations of the first level of abstraction formed through identification abstraction. It may then be asked what is common and specific to these stages. In that way an initial generalisation of a higher level of abstraction may be formed, which will represent a broader concept about reflection, i.e., one of reflection at the level of the living.

The genetic relation of the forms of reflection at different historical stages of the evolution of the external world is similar to the relation of the forms of the motion of matter, in the sense that simpler forms of motion and reflection do not disappear with the rise of more complex ones, that the higher forms of motion and reflection cannot be reduced to more primitive ones, and so on. The universal and specific that will be fixed

in a generalisation about reflection at the level of the living will therefore contain characteristics that we have already met at the lowest stage (and level) of reflection, i.e. at the stage of reflection at the premental level.¹⁴

Thus, if we are dealing with a series of objects $A_1, A_2 \dots A_n$, genetically linked with one another and representing stages of development of one and the same phenomenon, their generalisation will include characteristics common to $A_1, A_2 \dots A_n$, but contained in primitive form in A_1 . It is thus characteristic of reflection at the level of the living that

(1) reality is reproduced in it; in some cases the reproduction may occur through physico-chemical processes taking place in the protoplasm of cells, in others at the level of higher nervous activity;

(2) reflection always has an anticipatory character;

(3) at certain stages the process of reflection presupposes direct interaction, contact of the reflecting organism and the object, and allowance in the first place for their similarities and differences;

(4) the process of reflection always presupposes use of coded semiotic systems.

Nominalist approaches to the analysis of knowledge have been extremely common in neopositivism in the twentieth century, but they have not provided opportunities of dealing satisfactorily with the problem of generalisation.

The epistemological basis of all forms of nominalism is the following approach to analysis of objects' properties. The things observed, material objects united in genera and species, only have something in common at the level of coarse, superficial analysis. The point is that what each object subject to identification has in common with other properties is only seemingly so; in fact each embodiment of the common property in a particular object is unique and individual. The 'common' property therefore breaks down, when it is embodied in separate objects, into a set of individual properties. That can also be said *mutatis mutandis* about the properties of properties, about the properties of the properties of properties, and so on *ad infinitum*.

In fact we can identify people by the property 'of having developed, articulated speech', 'of possessing upper and lower

extremities', 'of possessing a sense of humour', etc. But each person has a way of speaking proper to him alone, has special traits of the upper and lower extremities that differ in many respects, special displays of the sense of humour, and so on. The common properties thus break down into a set of individual properties, i.e. of separate objects. But perhaps these separate objects have something in common with one another? There is something in common, for example, in all the individual ways of speaking. But if we bring this something to light it may turn out to be an individual embodiment in individual ways of speaking, and so on.

That means, the nominalists conclude, that material objects have nothing in common. The common is manifested in knowledge because of inexact analysis of the nature of things. The common is arbitrary and is expressed only in words signifying genera and species of things. The nominalist conception can explain why different words mean different things, but it cannot explain why one and the same general word signifies various separate things, and why we employ generalisations, etc., and that is what constitutes its unsoundness.

§ 2. The Difficulties of Traditional Sensualist Theories of Abstraction and Generalisation and the Way to Overcome Them

The processes of the formation of initial conceptual generalisations on the historical, genetic, and ontological planes must be distinguished from their subsequent alteration and development. On the historical, genetic plane it is a matter of the forming of those initial generalisations (concepts) that develop for the first time in human experience and broaden our knowledge of the world. On the ontogenetic plane it is a matter of the individual's assimilation of the experience of mankind, including initial generalisations (concepts) already existing.

These planes are sometimes not distinguished sufficiently clearly in theories of generalisation and abstraction; the specific character of the historical aspects of generalisation and abstraction is not clarified. It was not, for example, in the theory of generalisation and abstraction developed by John Locke, which had a substantial influence on the subsequent development of philosophical and logical thought.

Locke argued as follows. Initial generalisations connected with singling out the universal and the specific in the set of objects *M* being analysed are made by discarding what is different in the elements of this set and retaining what is similar in them (including that similarity which appertains only to the elements *M*).¹⁵ By removing from the complex ideas 'Peter' and 'James', 'Anne' and 'Mary' that which is peculiar to them, and preserving what is similar in them, Locke wrote, we form a new general idea 'person'.

Of the complex ideas signified by the names 'man' and 'horse', leaving out but those particulars wherein they differ, and retaining only those wherein they agree, and of those making a new distinct complex idea, and giving the name 'animal' to it, one has a more general term, that comprehends with man, several other creatures.¹⁶

Locke's explanation of the individual's formation of such initial classes of generalisation (and of the concepts corresponding to them) as 'man', 'animal', etc., was not satisfactory because it presupposed preliminary knowledge of what was to be obtained as a result of the generalisation. We assume that we should make the choice of individuals that are men in order then to form a primary generalisation about men by retaining what is common and specific in these individuals and discarding what is different in them. This selection can be made, however, if one knows in advance which of the organisms around one must be studied so as to discover what is common and specific in them, i.e. one must be apprised of which of the animal organisms are men. In other words one must possess in advance the primary generalisation that one is trying to form.

How, in fact, are the initial classes of generalisation of the ordinary language and the concepts corresponding to them (primary generalisations) formed on the plane of ontogenesis? How is the circular logical procedures in this process, mentioned above, overcome?

The initial generalisations of the ordinary language are formed by including social experience in the individual act of cognition performed by the subject. The requisite initial generalisations and elementary concepts are formulated on the basis of the individual's active attitude to social experience, on the one hand, and of the same attitude to the object of cogni-

tion on the other. In this connection, Leontiev wrote:

As modern psychological and genetic-epistemological research has shown, inner, mental activity is not only a derivative of external, practical activity, but has the same structure in principle as practical activity.¹⁷

The functions of the inclusion of social experience and an active attitude to the process of cognising the object, described above, are satisfied by what is called *ostensive definition*. In its simplest form it functions as the ostensive definition of a term. Cornel Popa writes:

By *ostensive definition* a whole number of authors (W.E. Johnson, Bertrand Russell, Joseph M. Bochenski, Robert Robinson, D.P. Gorsky, and P. Cous) understand the method of establishing the sense of a linguistic expression by simultaneous pronunciation of words and pointing to the object indicated by them.

If I pronounce the word 'giraffe' in the presence of a companion (who, we suppose, has not hitherto known of the existence of this animal), when this companion, for example, a child, is looking closely at some animal in the zoo, this means that I am giving an ostensive definition of the term 'giraffe' to this perceiving subject.¹⁸

The child may thus have formed a simple conceptual generalisation. The class corresponding to the primary generalisation represents the meaning of the language expression of the term 'giraffe'. The elementary concept of giraffes forms the sense of the term 'giraffe'.¹⁹

In more complex cases word combinations of various complexity may be defined ostensively, including such propositions as 'to pour milk', 'to turn off the tap', and 'to construct an equilateral hexagon'. Effective assimilation of the symbolic expressions that are of fundamental importance for forming a child's language does not simply come about through the influence of a teacher but also presupposes active work by the child in regard to the object named by the adult. It is not my aim to examine the problematic of ostensive definitions.²⁰ On the basis of knowledge of a certain class of generalisations ostensively assimilated, we can also introduce other generalisations of a concept of ordinary language (and not just ordinary language) *verbally*, i.e. by means of verbal definitions based on words and word combinations mastered in the last analysis ostensively.

The process considered here of the forming of primary generalisations by ostensive definitions indicates how they can take shape in the individual's language. But that presupposes that these generalisations already exist in the language of humankind, which means that the mode of the forming of initial generalisations in the individual's language through ostensive definitions does not conclusively solve this problem, but only postpones solution. The problem of how that kind of generalisation is initially formed in the language of mankind has to be explained. In other words it is necessary to solve the problem of how the subject forms initial generalisations on the historical plane, on the plane of the social subject's phylogenetic development, i.e. how generalisations still unknown to humanity are formed.

If we employ Locke's procedure of the formation of initial conceptual generalisations on the historical, genetic plane, then the considerations that used to be formulated against it (when it was formulated on the plane of ontogenesis) remain valid; it turns out that to obtain knowledge about the general inherent in a certain set of studied objects, we must be able to single this set out preliminarily, i.e. know its general and specific characteristics. Knowledge of the universal thus proves to be preceding operation with individuals and therefore cannot be abstracted from them.

Husserl, and other philosophers of a Kantian hue, noted that if we selected individuals in order to deduce the general from them without employing a previously given idea of the general, we would be unable to form any concepts of them at all. This kind of vicious circle in the sensualist theories of abstraction that underlie the formation of initial generalisations was also noted and criticised by the German logician and philosopher von Sigwart, who wrote: 'to want to form an idea that way by abstraction is consequently to look for the spectacles that are on one's nose with the help precisely of those spectacles'.²¹

A German philosopher of the turn of the century, the Neokantian Cassirer, also criticised sensualism and empiricism on matters of abstraction and generalisation and tried to show that it was impossible to form scientifically and practically significant primary generalisations by means of the proposed sensu-

alist decisions, since not only was the choice of the individuals interesting us based on the general being singled out, but we also (and that, from his point of view, was the main point) did not single anything general out and did not use anything general to form classes of generalisation and the concepts corresponding to them. According to him we only choose the essential general by means of which a concept could be formed that constituted a unity of diversity that made sense.

If we (Cassirer wrote)... subsume cherries and meat under the group of red, juicy, edible objects, we thus do not get a valid, logical concept but a meaningless word association that does not signify anything, and is no good for comprehending the separate cases. It therefore turns out that the general formal precept alone is not sufficient in itself, but that on the contrary it tacitly falls back in all cases on another mental *criterion* for its completion.²²

According to him this criterion was provided by general, *a priori* categories of common sense that regulated the forming of the general (including general initial generalisations) when single facts were being studied. A mental act of identification itself underlay every generalising abstraction. It was a function of reasoning and 'itself has no direct sensual correlative to the contents compared'.²³ These were their own 'forms of consciousness'.²⁴

The Russian psychologist and logician Lange defended a similar position. From his point of view

things cannot be called different or similar... They are linked together in notions about logical relations of similarity and difference only for thinking, which unites comparable things.²⁵

Husserl developed a theory of the conditions of cognition on an idealist basis, according to which the generic community of objects was comprehended by means of a special 'thinking intention', a doctrine that included a theory of generalising abstraction.

The simple stressing of a partial content is not logically or epistemologically abstraction but the peculiar consciousness to denote that the specific unity is grasped on an intuitive basis.²⁶

To the methodological difficulties examined above can be added difficulties consisting in its being impossible, on the basis of induction, and relying on the identification abstraction

described above, to form concepts of sensually imperceptible properties like 'having such-and-such a value', 'being in such-and-such a number'.²⁷ Because of the metaphysical approach to tackling these difficulties of the Lockean type of theory of generalisation and abstraction, idealist opinions on this matter have been strengthened. The inability to give a scientific materialist interpretation of the historical process of the formation of natural number, relying on experience and social practice, led the famous mathematician Kronecker to an openly idealist point of view that the Lord God created whole numbers, and all the rest were the work of human hands.²⁸

Modern neopositivists (Carnap, Schlick, and others) have not specially examined the process of the forming of knowledge (concepts and elementary statements) on the historical plane, although it is precisely they who should have done so, since they had proclaimed their conception the philosophy of 'pure empiricism', and assigned the status of a certain fundamental basis of all experimental constructions to the elementary propositions of observation.

The difficulties connected with forming initial generalisations are resolved by concrete application of the methodological precepts of dialectical materialism on the role of social practice in knowledge (cognition), and on the inclusion of the historical in the logical, and of historical-genetic approaches in examination of the forming of the 'cellules' of knowledge that function as the primary, initial ones in humanity's experience.

In this connection Lenin wrote that

the standpoint of life, of practice, should be first and fundamental in the theory of knowledge. And it inevitably leads to materialism, sweeping aside the endless fabrications of professional scholasticism.²⁹

and that

in the theory of knowledge, as in every other sphere of science, we must think dialectically, that is, we must not regard our knowledge as ready-made and unalterable, but must determine how *knowledge* emerges from *ignorance*, how incomplete, inexact knowledge becomes more complete and more exact.³⁰

Karl Marx had noted that the process of people's cognition of the surrounding world does not begin with theoretical comprehension of it. People start with actively affecting the reality around them, assimilating objects of the external world

by actions, and in that way satisfy their needs. They thus begin with production.

The property of these things 'to satisfy needs' is stamped on man's brain through repetition of these processes, and men, like animals, also learn 'theoretically' to distinguish the external things that serve to satisfy their needs. At a certain level of further development (Marx continued) when, meanwhile, their needs and the actions through which they are satisfied have increased and developed further, these empirical things distinguished from the rest of the external world are put verbally into a whole class... But this verbal denotation only fully expresses as the notion that which repeated confirmation by experience has created.³¹

While consistently following the historical point of view, the founders of Marxism-Leninism did not divorce man from the animal kingdom and the higher animals. Engels' ideas on the origin of man from anthropoid apes, and of the existence of rudiments of rational activity in animals, are of great value in that respect for modern science. Halperin, basing himself on those ideas when analysing the processes of generalisation in higher animals, writes:

The object with which the animal interacts has to function in a generalised way: if it is 'an enemy', it must not be an individual enemy but at least an enemy of the species; if it is a quarry, it must also function, so to speak, in a general way; if a wolf only pounced on a sheep exactly like one eaten before, and ignored all other sheep, such a 'pedantic' wolf would soon become the victim of natural selection.³²

The founders of Marxism stressed that the construction of a scientific theory was preceded by a long period of analysis of a certain field of reality, the formation of certain abstractions, and initial generalisations (concepts), the formation of statements, generalising facts in ideas, and so on. Analysis of the concrete in reality thus precedes the construction of a theory. At the level of a theory we obtain concrete knowledge in the sense that it reflects the concrete in reality in its essential, natural connections.

It is therefore not fortuitous that many theories are expounded as a process of ascent from the abstract to the concrete. The exposition of theoretical knowledge for teaching purposes is thus often inductive. In my opinion that is the most fruitful

path. Inductive methods of expounding material, in which there is consistent generalisation of concepts, are those most favourable for students' active assimilation of the material. In that sense the inductive method has the advantage over the deductive. It is therefore difficult to resist recalling the advice Hilbert gave to Weyl: 'Begin with the simplest examples'.³³

The process of identifying the objects studied so as to form concepts of them can be realised in various ways. It can be done by comparing objects with one another. This process is mainly proper to the procedures of forming concepts during a student's assimilation of the accumulated experience of mankind.

Another way of generalising studied objects in order to form concepts is to identify them by the use of a selected *standard*. In this case the process of comparing the studied objects with one another is mediated by an equality-type relation *R* between the selected standard and the other studied objects.

Comparison of the objects studied by singling out their relation to a standard is a process linked with material activity and social practice, and with taking into account the essential relationships in which the studied objects are found. This process not only corresponds to certain procedures for forming concepts employed in logic and the methodology of science, but also corresponds to historical facts witnessing to the formation of concepts. Davydov, with that aspect of the matter in mind, writes:

A real generalisation is not made by a formal comparison but by analysis of the system, bringing out the universal, form-deriving significance of a certain *relation* in it, (my italics—*D.G.*).³⁴

The procedures set out in the next section for forming initial generalisations (concepts) can be regarded as a concretisation of the general dialectical-materialist methodological precepts formulated above.

§ 3. Definitions by Abstraction. Their Generalisation, and the Dialectical-Materialist Conception of the Formation of Initial Generalisations (Concepts)

The philosophical, methodological precepts of dialectical materialism become adequate for overcoming the difficulties

discussed above only when they are organically included in the process of investigating a subject and creating a corresponding theory. Let me try to concretise that by applying definitions by abstraction to the forming of fundamental concepts (initial generalisations) of science that reflect the directly sensually imperceptible properties of objects.³⁵ The methodological significance of the theory of definition by abstraction consists above all in its indicating how the sensually imperceptible properties of separate objects are singled out in pure form (and at the same time generalised in appropriate concepts). Definition by abstraction can be employed as well to generalise studied objects by their sensually perceivable properties and the formation of appropriate concepts.

The directly sensually imperceptible properties of objects are singled out by establishing *equality-type relations* R between the studied objects. These are binary relations between objects x and y from domain D that simultaneously satisfy the following axioms:

(1) relation R is symmetrical in domain D :

$$xRy \supset yRx;$$

(2) relation R is transitive in D :

$$\bullet \quad xRy \wedge yRz \supset xRz;$$

(3) relation R is reflexive in D :

$$xRx.$$

Examples of equality-type relations are the following: ('is equal to', 'is like', 'is numerically equal to', 'is equivalent to', 'is exchangeable for' (on the market), 'is simultaneous with', 'is parallel to', etc., since substitution of them for R in axioms (with simultaneous replacement of the variables x , y , and z by concrete elements of the respective domain) will give us true propositions.³⁶

Axiom (1) of equality-type relations is formulated as follows: if a pair of objects x and y satisfies relation R , then the latter is also satisfied by the pair of objects y and x . The axiom of transitivity (2) means: if relation R satisfies the pairs of objects x and y , and y and z , it also satisfies the pair x and z .

Axioms (1) and (2) are used in the general case to establish equality-type relations between *different* objects in a certain abstraction (generalisation) interval.

The axiom of reflexiveness (3) means: object x is in equality-type relation R with itself. When we study objects in the conditions of a certain stoppage of motion (i.e. in conditions of a certain abstraction and idealisation), this axiom is valid.

If we add a fourth axiom $A(x) \supset (x=y \supset A(y))$ to axioms (1) to (3), we get a definition of a relation of identity (equality) signified by the mathematical sign $=$.

Axiom (4) is formulated thus: if x possesses properties A , then y has the same properties, since $x=y$. This axiom is a variant of the Leibnizian definition of identity ('Leibniz's law'), if A is all the properties belonging to both x and y simultaneously. Equality-type relations R are often called *equivalence* relations.

Let us first examine a certain elementary domain.

Let us assume that all its objects are equivalent in pairs, i.e. are identical with one another by the equivalence relation R , and coincide with a certain set (class) M_1 . Then, by employing the *principle of comprehension*, which consists in its being possible to correlate each set with the property by which it is formed ($x \in M \equiv P(x)$), we can correlate set M_1 , coinciding with D_1 , with property P_1 . On the basis of relation R and the principle of comprehension, it turns out that we generalise objects of domain D_1 , coinciding with set M_1 , in the concept $xP_1(x)$.

Furthermore, let axioms (1) to (3), characterising relation R and the condition $\forall x \exists y (xRy)$ be at our disposal: 'for an object of a domain there is one which is in relation R to it'. Our elementary domain D_1 , which coincides with M_1 , will include objects $O_1, O_2, O_3 \dots O_n$. On that basis it can be shown that all the objects of this domain are equivalent in pairs (Theorem 1).

By removing the universal quantifier and existence quantifier from formula $\forall x \exists y (xRy)$, we can get O_jRO_1 , in which $j \geq 1$, or O_iRO_1 (which is the same as O_jRO_1). In other words, we get: $O_1RO_1, O_2RO_1, O_3RO_1 \dots O_nRO_1$.

Thus, the following are given:

O_jRO_1 (1), O_iRO_1 (2), and axioms (1) to (3)

It is required to prove that O_jRO_i .

(1) From axiom (1) and O_iRO_1 , by substituting O_i for x and

O_1 for y , and employing *modus ponens*, we can obtain O_1ROi (3).

(2) $O_jRO_1 \wedge O_1ROi$ can be obtained from O_jRO_1 and O_1ROi (using the rule of introduction of a conjunction in formulas (1) and (3)).

(3) From axiom (2), making the proper substitution in it, and $O_jRO_1 \wedge O_1ROi$, we can get O_jROi (Q.E.D.)

This means that, in the conditions formulated above, the objects of our domain are equivalent in pairs, or there is a relationship $\forall x \forall y (xRy)$. All the objects of domain D_1 thus prove to be identified and generalised by relation R and form set M_1 .

On the basis of proved theorem (1) it can be said, by applying the principle of comprehension to it, that all the elements of set M_1 will have property P_1 . Now let us examine domain D (more complex in composition) as a general case.

An equality-type relation (one of equivalence) breaks domain D down into classes $M_1, M_2, M_3 \dots M_k$, which do not have any elements in common, as soon as objects are found in D that are not in relation R to objects $O_1, O_2 \dots O_n$ (the *multiplicative principle* formulated by Bertrand Russell). Let us prove that this is so.

Let us agree that there is an object O_1^1 in domain D , that is not in relation R to any of the objects $O_1, O_2 \dots O_n$. Then it will not belong to class M_1 . But if there are other objects $O_2^1, O_3^1 \dots O_n^1$, that are in relation R to O_1^1 , then, by virtue of the theorem proved above, they will constitute a class M_2 . No one element of class M_2 will be in M_1 , since in that case element O_1^1 would have to be in M_1 , which contradicts the condition.

What has been said above relates to all objects O_1^2, O_1^3 , etc., met in domain D that are not in relation R to $O_1 \dots O_n$, or to $O_1^1 \dots O_n^1$.

This means that domain D is broken down by relation R into sets $M_1, M_2, M_3 \dots M_k$, which do not have elements in common. The elements of each of these sets are generalised respectively by the properties $P_1, P_2, P_3 \dots P_k$. Each of the sets $M_1, M_2, M_3 \dots M_k$ can be represented, according to the *principle of abstraction*, by their separate elements.

In order to explain this, let us consider a concrete example.

Assume that domain D consists of very different commodities. Let commodities $O_1, O_2, O_3 \dots O_n$ function in a relation of

mutual exchangeability R . Then they will be 'stuck together' by some property common to them: being directly exchangeable for one another they will be equated with one another by relation R . The property in which they are equated with one another can be called the *value of precisely these goods*. We thus discover a common property of 'having such-and-such a value' in commodities $O_1, O_2, O_3 \dots O_n$. A set of goods M_1 will correspond to this property. (1) This property will be distinguished by the fact that it is sensually imperceptible. Karl Marx pointed out that when concepts like value are being analysed, 'neither microscopes nor chemical reagents are of use. The force of abstraction must replace both.'³⁷ (2) Its singling out presupposes practical operating with goods, i.e. the establishing of an equality-type relation between directly exchangeable commodities.

Other commodities of domain D , namely goods $O_1^1, O_2^1, O_3^1 \dots O_n^1$ may also be in a relation of direct exchangeability with one another. But none of these goods O_i^1 will thereby be exchangeable for the previously considered goods O_i . A sensually imperceptible property 'of having such-and-such a value' may also be discovered in the O_i^1 goods, but this value will differ in size from that contained in each of the O_i goods. This property will correspond to a set of goods M_2 . That also means that the sets M_1 and M_2 will not contain common elements.

In the conditions described above sets $M_3 \dots M_k$ will also prove not to have elements in common with one another, and with the M_1 and M_2 sets of goods. Comparing sets $M_1, M_2, M_3 \dots M_k$, which have no elements in common, it may be asked what they do have in common; and that is that they are bearers of value (which all goods of domain D possess, irrespective of which set they belong to). *What is common in each of the classes united by a relation of direct exchangeability with one another will thus be the property of value P .* We can thus form a concept of value in general, viz., $xP(x)$. Set M corresponds to value property P , and represents an aggregate (pooling) of sets $M_1, M_2, M_3 \dots M_k$:

$$M = M_1 \cup M_2 \cup \dots \cup M_k.$$

Value property P is what is common in its various modifications, i.e. in properties $P_1, P_2, P_3 \dots P_k$. When we single out what is common in them (and so generalise the modifications of pro-

perty P) we reduce the content of the concept (in this case the concept of value: from concrete values of groups of mutually exchangeable commodities we pass to value in general). The range of the concept of value is correspondingly expanded; we make the transition from separate classes $M_1, M_2, M_3 \dots M_k$ of mutually exchangeable commodities to all classes of commodities exchangeable for one another, i.e. to their unification.

The theory of definitions by abstraction was originally developed by mathematicians in the context of the theory of sets (above all by George Cantor, but also by Walter Dubislav and Hermann Weyl, and others), and was employed to develop a definition of *cardinal natural number*. Thus, in the first stage of the procedure I have described, by using definition by abstraction we could have made such abstractions as the numbers 'five', 'ten', 'twenty', etc. The words 'five', 'ten', 'twenty', could have been introduced as the names respectively of the set of aggregates numerically equal to the number of fingers of one hand, the set numerically equal to the fingers of both hands, and so on. This means that domain D would contain sets $M_1, M_2, M_3 \dots M_k$, not intersecting one another in pairs, i.e. not having elements in common. We would thus originally generalise different classes numerically equal to one another. These generalisations would be built up from corresponding *standard* sets: the fingers of one hand, the fingers of two hands, and so on. Sets numerically equal to these standard sets would thus be selected, i.e. sets in a relation of *one-to-one correspondence* (in relation R) to the standard sets.

So, having selected sets numerically equal to the fingers of one hand (the set of 'petals of a buttercup', the set of the 'vertices of a pentagon', the set of 'letters in the word "digit"', etc.), we could single out what is common to all of them, i.e. generalise these objects and form a concept $xP_1(x)$ from them, viz., 'of numbering five'. The range of this concept will consist of sets numerically equal to the fingers of one hand. The set M_1 (its elements would be the sets: 'fingers of the hand', 'petals of a buttercup', etc.) would thus correspond to the content of the concept $xP_1(x)$. We could form sets $M_2, M_3 \dots M_k$ in the same way for other standard sets, each of which would have an individual cardinal number.

The individual characteristics associated with the dif-

ference of the sets $M_1, M_2 \dots M_k$, corresponding to different numbers, can now be disregarded, i.e. we can generalise the objects $M_1, M_2 \dots M_k$, and single out what they have in common. What is common in sets of different cardinal number characterised by different numbers (which is manifested in the difference in their names) will be the *cardinal number in general*.

The *essence of definition by abstraction* is that we break domain D down by an equality-type relation R into classes of objects (elements) identical with one another that have no elements in common. Within these classes $M_1 \dots M_k$ the elements are 'stuck together' by a property common to them (abstraction by identification, or generalisation) on the basis of the indistinguishableness of their other properties (indiscernibility abstraction). Classes of objects $M_1 \dots M_k$ identical in themselves can be identified in turn, and generalised in a new, more abstract class X , to which classes $M_1 \dots M_k$ are related as its elements.

The first attempts in the Soviet Union to employ definitions by abstraction outside mathematics were made by Yanovskaya in a work published in 1936.³⁸ She applied them in the field of political economy, and in particular to the forming of the concept of value.³⁹ But her analysis of the singling out of properties and the forming of concepts of them was limited to the domain of sensually imperceptible properties: 'to be such-and-such a number', 'to have such-and-such a value', 'to have such-and-such a probability', and so on. Furthermore she demonstrated the unity of the historical and logical methods in the forming of concepts of quantitative (cardinal) number and value.

Let me turn once more to the point that, in singling out a property P , we employ a relation R , which do not depend on one another. The properties of 'having such-and-such a number', 'having such-and-such a value', 'having such-and-such a probability', 'having such-and-such a form' (geometrical), 'having such-and-such a weight', etc., are distinguished respectively through the following equality-type relations: 'equinumerousness', 'equal value (exchange)', 'equiprobability', 'equiform (likeness)', 'equiponderability', etc. But the property of 'having such-and-such a number', for example, does not, in

a certain sense, depend on the relation of equinumerousness (of a one-to-one correspondence), although the linguistic expressions 'having such-and-such a number', and 'equinumerousness' have a similar meaning. The same can be said of all the other pairs P and R .

The sense of an equality-type relation R is realised through a certain *standard* and the mode of selecting objects similar to it. *The properties P , formed through definition by abstraction are already thought of as independent of the standard and the mode of operating with it.* They are the results of abstraction. Through further application of abstraction by identification we can pass to an abstraction of a higher order.

Bearing in mind what has been said, I suggest that *definitions by abstraction can also be extended to cases of the singling out of sensually perceptible properties*, such as 'being white', 'having such-and-such a shape (of real objects)', 'having such-and-such a weight', 'having such-and-such a loudness', 'having such-and-such a remoteness', etc.

The equality-type relation R here will be a relation of *concrete identity* ' x is identical in some sensually perceivable property P with y '. It possesses properties of symmetry, transitivity, and reflexivity.

A relation of concrete (partial) identity (similarity, sameness) refers to equality (identity) as a relation of the interchangeability of objects only in certain cases to their interchangeability in any case, i.e. to their full coincidence. Such an interpretation of identity is common.¹⁰ Objects equally identical with the standard will be selected by means of identity and on the basis of a certain standard, moreover, at the level of intuition. The *equal identity* of objects with the chosen standard in property P must be distinguished from the *property P* already abstracted, which we possess at the level of thought irrespective of the standard.

Thus, if we take snow as the standard, we can form a class of equally white objects on the basis of relation R (' x is identical in some colour property with y '). What is common in all identically white objects will obviously be the property of colouring, which is given the name of 'being white' or 'white'. Insofar as we already distinguish objects from one another intuitively by their coloration, relation R breaks domain D

of objects considered from the angle of their coloration into sets that have no common elements, i.e. into classes of equally red, equally blue, equally yellow, and such-like objects. By applying abstraction by identification to these classes we can form an abstraction of a higher order, viz., 'colour'.

By relying directly on the evidence of our sense organs, on intuition, and on the chosen standard, we can correspondingly form classes of equally heavy objects (identifying them with the standard by muscular sensation), equally loud sounds, equiform objects, and so on, and then form abstractions of a higher level such as 'weight', 'loudness', 'shape' (of real objects), and so on. Through equifunctionality (relation *R*) we can distinguish classes of objects that may in practice replace one another.⁴¹

The singling out of sensually perceptible properties is a more complicated process than that of distinguishing sensually imperceptible properties. The latter is done by quite precise procedures admitting of an algorithmic description (compare the relation of one-to-one correspondence that is used to form cardinal numbers and the relation of the exchangeability of goods on the market, by which those of equal value are distinguished). The properties singled out in these cases have a quantitative character; they are evaluated by operations of counting and measuring. Sensually perceptible properties have a qualitative character and are singled out by the subject's cognitive activity realised on an *intuitive basis*, which includes much that is subjective: for example, some objects may appear equal in colouring to some subjects (or correspondingly by weight), but not to others. These subjective elements are largely eliminated in the historical process of cognition based on practice.

Just as the essence of the concept of value was only brought out in the mid-eighteenth century, the essence of many concepts of sensually perceptible properties was disclosed at the stage of developed natural science (for example, the concept of colour as a property of bodies to excite a certain visual sensation in accordance with the spectral structure of the radiation reflected by it or emitted by it took shape comparatively recently).

The approach considered above to the forming of concepts that are initial generalisations of sensually perceptible properties

on the basis of the theory of definition by abstraction includes people's practical, active work in selecting objects that have the same characteristics on the basis of some *standard* frequently encountered in experience. The method of forming concepts considered above indicates how knowledge arises from ignorance, how names are introduced for classes of objects, and is consequently orientated on *genetic approaches*.

The 'vicious circles' in the theory of knowledge that cannot be avoided when one adopts positions of Lockean contemplative sensualism disappear with such an approach. Thus we do not have to have an idea of white in advance in order to select white objects and form a concept of white objects, and it is not necessary to have a previous idea of a human being in order to distinguish people from animals and form a concept of man. When, in the course of practical activity, we choose a standard a^* , important for and relevant to that activity, and directly representing a property P that interests us (which need not be specific to a^*), we then choose objects that are similar to this standard in properties P (in this case the subject's capacity to identify and differentiate objects by their properties is assumed). We thus generalise individual objects, selected according to certain properties, in elementary concepts, i.e. the x 's that possess the properties P of a certain standard a^* .

Constant comparison of the standard P (a^*) and the chosen objects possessing property P is important in the forming of concepts that are elementary generalisations of individuals in thought. In some cases a standard has to be replaced by another, more typical and characteristic. In this case the words of Karl Marx are relevant:

In a sort of way, it is with man as with commodities. Since he comes into the world neither with a looking glass in his hand, nor as a Fichtean philosopher, to whom 'I am I' is sufficient, man first sees and recognises himself in other men. Peter only establishes his own identity as a man by first comparing himself with Paul as being of like kind. And thereby Paul, just as he stands in his Pauline personality, becomes to Peter the type of the genus *Homo*.⁴²

The point here is the formation of concepts of properties of objects sensually perceivable by man.

The standard approach associated with the choice of objects

identified in certain sensually perceivable characteristics P with standard a^* plays an immense role in the forming of initial generalisations in science.

Let me adduce some illustrations in support of this thesis. Oxygen was originally discovered by the Swedish apothecary C.W. Scheele, but his work was not published and had no influence on subsequent research. Later the English scientist and divine Joseph Priestley announced the discovery of oxygen. He had

collected the gas released by heated red oxide of mercury as one item in a prolonged normal investigation of the 'airs' evolved by a large number of solid substances. In 1774 he identified the gas thus produced as nitrous oxide and in 1775, led by further tests, as common air with less than its usual quantity of phlogiston.¹³

The result of the first experiment thus served as the standard for identifying it with the results of subsequent experiments. The work begun by Priestley was continued by Lavoisier. At the beginning of 1775 Lavoisier came to the conclusion that the gas he had discovered was air fit to breathe. 'By 1777 ... Lavoisier had concluded that the gas was a distinct species.'¹⁴

The description of oxygen (discovered by Lavoisier, Priestley, and Scheele) was made from separate experiments. In this case it was assumed that object a^* (later called oxygen) with the genetic characteristics P , which played the role of standard, would be discovered in subsequent experiments carried out in the same way. The assumption was later validated. The standard object, originally described by the mode of obtaining it (i.e. operationally genetic), was later characterised in the course of further study, through its own properties, i.e. by its propriety, and became the standard P (a^*) for identifying all gases with property P with it.

Here we encounter a situation in which there is use of standards of two levels: (a) the operational-genetic, and (b) the property one.¹⁵

In the experimental sciences initial generalisations very often have an operational-genetic character at first. Only later do they acquire a property character, i.e. description of the modes of obtaining and of measuring the standard a^* is supplemented by description of its specific properties P (a^*).

X-rays were discovered in an analogous way. They were first discovered in a definite experiment and characterised initially by the way they were obtained; their properties were described later. Roentgen discovered the action of these rays on a photographic plate and their ionisation of air, showed that they had not been reflected from a mirror surface, and discovered the laws of their absorption and the connection of the absorption with the density of the substance. Sometime later their polarisation and diffraction were discovered. Here, too, there was a consecutive forming of standards of two levels; viz. on operational-genetic and property levels.

§ 4. Abstraction by Identification as a Means of Generalising Scientific Theories

Scientific theories are often developed through the applications of procedures of identification to them. Assume that certain sets of objects $A_1, A_2, A_3 \dots A_n$ relating to one and the same subject domain are being studied separately at first. Their specific characteristics will be thereby revealed. Then the results of studying $A_1, A_2, A_3 \dots A_n$ will be generalised. Certain common properties of them are discovered in sets $A_1, A_2, A_3 \dots A_n$. Then, by abstracting from the specific elements of each of the sets $A_1, A_2, A_3 \dots A_n$, certain general relations are formulated as a function between elements of the sets and elements of the subsets of the sets, and so a generalised theory is constructed. Each set from $A_1, A_2, A_3 \dots A_n$ proves to belong to a set $A_1 \cup A_2 \cup A_3 \cup \dots \cup A_n$.

Thus, through study of the linguistics of German, English, Danish, and other languages, they can be united in a group of Germanic languages. The task of identifying and generalising classes $A_1, A_2 \dots A_n$ is done by formulating characteristics and laws common to each of the classes belonging to their association: $A_1 \cup A_2 \cup \dots \cup A_n$.

Laws of this kind, at the level of empirical knowledge, may be phenomenological or descriptive. At the theoretical level so-called *fundamental theoretical schemes* correspond to them (or at least to some of them). These laws can also be formulated as ones whose intrinsic characteristic is their explanatory function. Certain *pictures of the world* correspond to the

aggregates of these laws.⁴⁶

Feynman, Leighton, and Sands write:

At first the phenomena of nature were roughly divided into classes, like heat, electricity, mechanics, magnetism, properties of substances, chemical phenomena, light or optics, x-rays, nuclear physics, gravitation, meson phenomena, etc. However, the aim is to see *complete nature* as different aspects of *one set* of phenomena. That is the problem in basic, theoretical physics today—to find the laws behind experiment; to amalgamate these classes. Historically, we have always been able to amalgamate them, but as time goes on new things are found. We were amalgamating very well, when all of a sudden x-rays were found. Then we amalgamated some more, and mesons were found.⁴⁷

Here, by 'amalgamation' is meant the identification and generalisation of certain classes $A_1, A_2 \dots A_n$ in a certain general class by establishing general fundamental theoretical laws and general interpretational pictures of the world. As examples of fundamental generalisations ('amalgamations') Feynman *et al.* cite the reduction of heat to mechanics, when thermal phenomena and temperature effects were explained by means of the laws of mechanics.

Another tremendous amalgamation was the discovery of the relation between electricity, magnetism, and light, which were found to be different aspects of the same thing, which we call today the *electromagnetic field*. Another amalgamation is the unification of chemical phenomena, the various properties of various substances, and the behaviour of atomic particles, which is in the *quantum mechanics of chemistry*.⁴⁸

The fundamental theory of physics—classical mechanics—was itself formulated as a generalisation

of such forms of mechanical motion as the swing of a pendulum, the free fall of bodies, the movement of bodies on an inclined plane, the motion of the planets (Kepler's laws). A similar situation is observable in the history of thermodynamics and classical electrodynamics, in which the separate aspects of the studied processes were reflected in a developed chain of partial theoretical schemes and laws long before the first generalising theories of these departments of physics were constructed.⁴⁹

Kepler's first law was originally formulated in relation only to the motion of the planets around the sun: every planet moves

in an ellipse, at one of whose foci the Sun is situated. Later it was made clear that other celestial bodies could also describe conic sections—circles, ellipses, parabolas, and hyperbolas—around the Sun under the effect of its attraction. On that basis the original formulation of Kepler's first law was generalised: a body moving about the Sun describes a conic section, at one of the foci of which the Sun is situated. This was a generalisation of a broader range of astronomical objects.

The formulation of differential equations, which describe a very broad class of oscillations of a different physical nature demonstrates the objective similarity and unity of physical processes. In that connection Lenin wrote: 'The unity of nature is revealed in the "astonishing analogy" between the differential equations of the various realms of phenomena.'⁵⁰

As a means of generalising scientific theories abstraction by identification is sometimes closely linked with subsequent extrapolation of the generalisations obtained. In the eighteenth century, for instance, and in the early nineteenth century, the divisions of physics were intensively developed on the basis of Newton's mechanics (Euler, l'Alembert, Bernoulli), for which abstraction by identification was widely employed. Lagrange formulated the equations of mechanics in so generalised a form in his *Mécanique analytique* as to permit of their extrapolation (with corresponding interpretation of the variables in the equations) in other fields as well (e.g. hydraulics).

The method based on uniting two (or more) theories by singling out in them the essential interactions of a general and fundamental character between the objects of the respective fields borders closely on that of generalising scientific theories described above. In this case, too, abstraction by identification is employed. But it is sometimes not so much the process of experimental discovery of the interactions of realms of objects, (previously thought to be unconnected) that is basic, as the process of creating appropriate idealisations that permit these interactions to be described mathematically in a general form. The classical example of this is Maxwell's creation of the theory of electricity and magnetism on the basis of the experiments of Oersted, Rowland, and Faraday. That generalisation was linked with the discovery of an essentially new subject domain, the electromagnetic field. It was found that it was

not the charges or the particles per se that were essential for description of the physical phenomena but the field in the space between them. The results obtained in the course of developing the former theories were described and explained in the new language of the theory of the electromagnetic field.

The uniting of two laws in the theory of relativity—those of the conservation of mass and the conservation of energy—in a more general one, that of the conservation of mass-energy, is an example of such generalisation. It became possible owing to the discovery of profound internal interrelations between mass and energy. The former opposing of mass and energy proved to be relative. It became clear that a material object possessing mass m also had an energy $E = mc^2$, and that a material object possessing energy E , on the contrary, had a mass

$$m = \frac{E}{c^2}$$

That generalisation could not have been obtained inductively like, for example, the law of the transition of mechanical energy into heat, i.e. through study of separate instances of this transition and subsequent generalisation and explanation of the phenomenon. To unite the conservation laws in a generalised law expressed by the formula $E = mc^2$, it required a change of existing views both of the world and of the laws of mechanics.

There are several other forms of generalisation in scientific practice. Assume that we have a rule P_1 for a domain O_1 and a rule P_2 for a domain O_2 . By means of a special form of identifying O_1 and O_2 we can generalise rule P_2 as well for domain O_1 . For the class of explicit real definitions, for instance, the so-called rule of commensurability or proportionality has been formulated: 'the definiens must be equivalent to the definiendum'⁵¹ (1).

A corresponding rule can be formulated for nominal definitions as the rule of interchangeability: in definitions of the type $Dfd \equiv Dfn$, the terms Dfd and Dfn can be substituted for one another in any standard context (2). Then, in order to generalise rule (2) for the domain of real definitions, the latter can be translated into nominal definitions. Such a translation is permissible since the information of interest to us in the real definition proves to be identical with the infor-

mation contained in the corresponding nominal definition.⁵²

Abstraction by identification is also employed to comprehend the historical development of the generality of statements. A certain single statement is usually formulated initially: a certain characteristic P is affirmed about the single subject of the statement and is recorded in its predicate ('this S is P ') (1). Then, through identification (generalisation) of a number of single objects in a certain class S_1 , a general statement 'all S_1 are P ' (or rather 'each element of class S_1 possesses property P ') (2) is formulated. During further investigation on the basis of applying abstraction by identification (generalisation), a class S_2 may be created that includes S_1 , as a proper part of it. In this case a statement 'all S_2 are P ' is formulated about class S_2 (or rather 'each element of class S_2 possesses property P ') (3). This way of obtaining more and more general statements through abstraction by identification of the objects recorded in the subject of the statement is often followed during the development of knowledge. In this case the property may also be generalised (sometimes only in certain final stages of the development of the knowledge).

Frederick Engels cited the following example when addressing himself to analysis of the problems of dialectical logic. The fact that friction produces heat was already known in practice, in a certain sense, to prehistoric man; he obtained fire by friction, and warmed the cold parts of his body by rubbing them. Only later was this formulated in an explicit statement: '*Friction is a source of heat*'⁵³ (1), which recorded a certain regular connection of phenomena.

Still further thousands of years passed until, in 1842 Mayer, Joule, and Colding ... formulated the judgement: *all mechanical motion is capable of being converted into heat by means of friction*.⁵⁴ (2)

Three years later J.R. Mayer was able

to raise the judgement of subsumption to the level at which it now stands.

Any form of motion, under conditions fixed for each case, is both able and compelled to undergo transformation, directly or indirectly, into any other form of motion.⁵⁵ (3)

In statement (3) the predicate P is generalised: here P is not the predicate 'is a source of heat' (or 'is converted into heat',

which is the same thing), but 'is convertible, directly or indirectly, into any other form of motion'.

By comparing statements (1) to (3) the historically realised generalising of knowledge is brought out: from the individual or singular through the particular to the universal.

We can regard the first judgement (Engels wrote) as that of individuality; the isolated fact that friction produces heat is registered. The second judgement is that of particularity: a special form of motion, mechanical motion, exhibits the property, under special conditions (through friction), of passing into another special form of motion, viz., heat. The third judgement is that of universality: any form of motion proves able and compelled to undergo transformation into any other form of motion.⁵⁶

§ 5. The Generalisation of Scientific Theories through Concretisation of the Initial Subject Field and Imposition of Certain Limitations on It

This form of the generalisation of scientific theories is characterised by the following elements:

(1) New specifications are introduced in respect of the components of the previous subject.

(2) The field of the objects studied by the new theory is broadened through its *concretisation*, i.e. the bringing out of new properties of the field and new relations between its components through the specifications introduced, rather than through the inclusion of other fields in it lying outside its former extensional sphere.

(3) Whereas, in the cases I earlier examined, the theory being generalised functions in relation to an already generalised one as a partial case of the latter, in this form of generalisation the theory being generalised is a *limiting case* of the already generalised one.

(4) The theory being generalised and the already generalised theory are linked on the basis of the *principle of correspondence*, which can be formulated as follows: the laws of the generalised theory pass asymptotically into laws of the initial theory with some specific parameter characteristic of this field of phenomena tending to the appropriate limit.

Let us look at some examples of the generalisation of theories

with a limiting transition.⁵⁷ The slow motions that we operate with in normal conditions are distinguished intuitively; they are those that Newton's mechanics deals with. At the same time there are also the fast movements that we meet in electrodynamics, but these are not so fast if we have infinite velocities in mind. But this difference between low and high velocities acquired a concrete, definite sense when a limiting velocity—the velocity of light, equal to 300,000 km/sec.—was introduced into physics from experiment.

At the turn of the century a contradiction was discovered in physics between the laws of electrodynamics and the Galilean principle of relativity (they proved to be non-invariant relative to the Galilean transformations). Michelson's experiment, however, confirmed the principle of relativity; the motion of Earth did not affect light phenomena.

How was the principle of relativity to be brought into agreement with the experimentally confirmed laws of electrodynamics? To do that Einstein had to alter the concept of simultaneity, and to reject notions about the independence of the size of a body from the velocity of its motion (i.e. the ideas on which Galileo had based himself when formulating his principle of relativity). Ultimately Einstein rejected the concepts of absolute time and space introduced by Newton.

That made it possible to reconcile the principle of relativity with the laws of electrodynamics and to apply the Lorentz transformations to the transition from one inertial system to another. In that case, the laws of electrodynamics proved invariant relative to these transformations. But that led to a contradiction between the principle of relativity and the laws of Newtonian mechanics confirmed by experience; it was discovered that Newton's equations were non-invariant relative to the Lorentz transformations. Then Einstein reformulated the laws of classical mechanics so that they became invariant relative to the Lorentz transformation equations.

So Einstein's relativistic mechanics was constructed.⁵⁸ But it was because the domain of the studied motions (the magnitudes of the velocities, simultaneity of events, and dependence between the size of bodies and the velocities of their motion) was concretised, that the concepts of the initial theory of classical mechanics could be regarded as certain limiting cases of

the generalised concepts of relativistic mechanics. The insignificant speeds that we mainly have to do with in the macro-world, for instance, can be represented as limiting velocities in relation to velocities close to that of light; the simultaneity of events in inertial systems moving at low velocities, relative to one another, as the limiting case of the simultaneity of systems moving at high velocities, and so on. The mathematical apparatus of relativistic mechanics was transformed in limiting cases into the apparatus of classical mechanics.

The generalisation of scientific theories that are inevitably based on a process of measurement, and on concretisation of a domain, is always linked with the imposition of certain limitations on its parameters. These limitations are connected, for example, with the rejection (albeit within certain limits) of the absolutely continuous and infinite. It is impossible to introduce quantitative specifications into the domain, and to concretise it, without establishing certain constants. It would be impossible to construct, say, a relativistic mechanics, to create an adequate mathematical apparatus for it, and to link the initial (classical) and generalised theory by the principle of correspondence, without introducing a maximum velocity, i.e. the speed of light. Only in relation to such a constant could motions have been classified as rapid and slow, and only on that basis could the limiting transition be made from the generalised theory to the initial one by fulfilling the relevant initial conditions. If the existence of infinitely high velocities were assumed, it would hardly be possible to concretise the field, and to draw effectively ascertainable boundaries between rapid and slow motions (as limiting cases of rapid ones).

Similarly quantum mechanics passes in the limiting case into classical mechanics. In this case the discreteness of the quantum of action h is disregarded because of its smallness compared with the dimensions of the macroworld. Schroedinger's equation is transformed into the Hamilton-Jacobi equation, which is the most common expression of the laws of classical mechanics. With the creation of quantum theory the studied field was also concretised by imposing certain limitations on some of its parameters. It was established that the exchange of energy between the emissive systems and the field took place discretely and was quantised. The notion of an infinitely

small portion in the exchange of energy had to be rejected, i.e. the idea of continuity. That also made it possible to draw boundaries effectively between small and large units of measurement in the transfer of energy, and to link quantum and classical mechanics by a limiting transition determined by the correspondence principle.

When Heisenberg was trying to construct a single generalised theory of matter (from which, on the basis of the correspondence principle, the properties of all known varieties of fields and particles were to be mathematically derived), he also attempted to use the idea of discontinuity. In addition to the constants c and h he introduced a limiting elementary length of the order of the diameter of an electron (10^{-13} cm), to which an elementary interval (quantum) of time of the order of 10^{-24} sec corresponded. The introduction of new limitations, connected with elimination of concepts of infinitely small time and space, into the parameters characterising the studied range of objects, opened up a possibility, in his opinion, of new, effective concretisations of the studied field, and of establishing the relationship between the initial theories and the generalised theory, i.e., a single theory of matter.

Quantum mechanics, for instance, passes asymptotically into classical mechanics in conditions when the insignificant value of the quantum of action h can be ignored by supposing its action to tend to zero ($h \rightarrow 0$). Relativistic mechanics is transformed into Newtonian mechanics at small velocities of motion, when the velocity of light can be considered infinitely great ($c \rightarrow \infty$). The general theory of relativity passes into the special theory in conditions of a gravitational field with the gravitational potentials g_{ik} tending to zero ($g_{ik} \rightarrow 0$ when $i=k$, and $g_{ik} \rightarrow 1$ when $i \neq k$), and with weak fields into classical mechanics, and so on.⁵⁹

The transition to generalised theories in which the process of measurement plays a significant role, is made by *concretising the studied field*, which means that certain classes of the studied system's characteristics (the domain's) begin to be differentiated in a fundamental way (and this difference is given a quantitative evaluation). In this case the concretisation is always connected with the imposition of certain limitations on the parameters characterising the system.

*When the correspondence principle makes it possible to elucidate the mechanism of the transition from the generalised theory to the initial, limiting one, the principle of the concretisation of the initial field and the imposition of limitations on it reveal the cognitive mechanisms that permit the transition from the initial theories to the generalised ones to be made. The principle of limitations can be formulated as follows: the transition from an old theory to a new one is made by introducing new postulates and limitations into the old theory with retention of all the limitations that were characteristic of the old theory.*⁶⁰

The principle of limitations can be formulated more concretely as the principle of concretisation of the original field and the imposition of limitations on it: the transition from the old theory to the new, generalised one is made through concretisation of the subject-matter studied by the old theory by introducing new postulates and limitations into the old theory while retaining its already existing ones.

The reverse transition, governed by the correspondence principle, is connected with elimination of the limitations introduced. In the examples considered we therefore regarded quantity c as 'tending to ∞ ', and quantity h as 'tending to 0'.

With the transition from an old theory to a new, generalised one, as Illarionov has pointed out, the class of the theory's experimentally tested propositions grows, which means that the class of propositions assumed without contradiction by the former theory but neither confirmed nor rejected by experience, is thereby reduced. Furthermore, the class of propositions that contradict such a developed theory grows; the number of possible propositions contradicting the theory increases through a reduction of the propositions admitted without contradiction in relation to the original theory.

Assume that a scientific theory contains a null class of experimentally confirmed propositions. Then, generally speaking, any proposition formulated will be permitted by this theory and consequently none of the propositions formulated will contradict it. When the class of general propositions accumulated by mankind and confirmed experimentally was quite small, the most varied fantastic statements were admitted as legitimate, and as not contradicting experience. With growth of the number

and generality of experimentally confirmed propositions, as a result of establishing the relationships between them, the number of propositions admitted without contradiction was reduced. That meant at the same time an increase in the number of propositions that contradicted the statements of science, i.e. the class of propositions excluded by science as contradicting the laws of nature was increased.

The introduction of new limitations into a theory, in addition to existing ones, narrows the class of propositions admitted without contradiction and broadens the class of experimentally confirmable ones. In pre-Newtonian mechanics, for instance, in a problem of a certain field of force and two points, any motion between the latter was admissible, and also motion along any trajectory. In Newton's mechanics only those motions were singled out as really feasible that satisfied a certain dynamic equation. This limitation becomes specially obvious when the principle of least action underlies mechanics, a principle that singles out only that trajectory from all others that obeys a definite condition.

But classical mechanics did not impose any limitations on the admissible velocities of the systems, or on their energy states and the transitions from one state to another. These limitations have been imposed by the theory of relativity and quantum mechanics. The classes of statements admitted without contradiction in regard to new theories with new limitations were reduced. These limitations, however, ensured rapid scientific progress, laid the basis for fruitful experimental research and for formulating new propositions that accorded with experience, and so led to an extension and generalisation of the class of experimentally verifiable propositions.

The problem of the correlation of competing theories, and of theories that replace one another during the development of knowledge, has been analysed by several writers in the past twenty years in connection with the thesis of their *incommensurability*. Some (primarily Paul K. Feyerabend, T.S. Kuhn, and Stephen Toulmin) maintain a thesis of their incommensurability in principle.

The main point that makes it possible to formulate the idea of the incommensurability of scientific theories is the absence of a language of observation neutral as regards theory that

would provide a basis for checking theories. The idea of neutrality of the language of observation, and of experience as the 'supreme judge' of theoretical constructs, firmly established in scientific knowledge, was erected into an absolute and subjectively interpreted by neopositivism, and then replaced by the above-mentioned authors by the idea of a 'theory-wise loading' of the language of observations, and of its dependence on the theoretical framework of the scientific discipline.⁶¹ In that connection it was argued that even a partial reduction of theoretical knowledge to the empirical, and definition of theoretical terms by means of empirical ones, were impossible. This led as well to a revision of the classical understanding of rationality, which ensured cognition and assimilation of the world on the basis of logic, reason, practice, and in general the modes of empirical checking (as distinct from religion, magic, etc.).

Affirmation of the absence of a neutral language of observations in relation to theory, of the 'theory-wise loading' of this language, led the writers mentioned to the conclusion that there could be no choice or contradictions between rival theories, since they were incommensurable. That conclusion allegedly stemmed from each theory's reflecting its vision of the world, and its not being describable by one and the same neutral language of observations in order to check them, since each had its own theoretically loaded language of observation.

The impossibility of contradictions between them was substantiated by the point that identical terms from which the propositions in the rival theories were constructed differed in fact in their meaning: they carried the impress of what made the theories different, and the 'theoretical loading' of the language of observation prevented elimination of these differences by checking the terms and propositions experimentally.

Difficulties arose in validating continuity in the development of knowledge and in the scientific theories succeeding one another, since the latter proved incommensurable. In relation to the theory (T_1) being generalised and the generalised theory (T_2), for instance, obtained from the principle of limitations (T_2 from T_1) and the correspondence principle (T_1 from T_2), these writers have tried to show that they are incommensurable,

and that there is no continuity between them. This point of view guided Kuhn, for example, in regard to classical and relativistic mechanics. He asserted that the laws deduced from the theory of relativity and employed for slow motions, were not in substance Newton's laws and only coincided with them in form (and not in substance). Kuhn also agreed that 'Newtonian dynamics has been derived from Einsteinian, subject to a few limiting conditions'.⁶²

Assume that propositions $E_1, E_2 \dots E_n$ register laws of the theory of relativity containing parameters that reflect coordinates of space and time, rest mass, etc. A new series of propositions $N_1, N_2 \dots N_n$ that are claimed to record the laws of Newtonian mechanics can be deduced from them by means of the apparatus of mathematics and logic and several supplementary propositions of the type of $v^2/c^2 \leq 1$.

Kuhn suggested, however, that although $N_1, N_2 \dots N_n$ are special cases of the corresponding laws of relativistic mechanics, they are no longer Newton's laws; the variables and parameters contained in them are interpreted as partial cases of the parameters and variables signifying Einsteinian space, time, and mass. They are interpreted as certain partial components of the Einsteinian structure of the world. The Newtonian paradigm was already finished with,

for in the passage to the limit it is not only the forms of the laws that have changed. Simultaneously we have had to alter the fundamental structural elements of which the universe to which they apply is composed.⁶³

In the passage from Newtonian mechanics to the theory of relativity a revolutionary transformation of views of the world takes place.

Just because it did not involve the introduction of additional objects or concepts, the transition from Newtonian to Einsteinian mechanics illustrates with particular clarity the scientific revolution as displacement of the conceptual network through which scientists view the world.⁶⁴

Kuhn thus threw doubt on continuity in the development of theories when this happened in a revolutionary way. Other writers deny the existence of a link between theories in general and consider transitions from one theory to another as possible by

way of purely psychological Gestalt-switching, and not on the principles mentioned above (of limitations and correspondence).

The arguments adduced by Kuhn to substantiate his thesis about the absence of continuity in the development of theories in a revolutionary way seem unconvincing to me.

Propositions $N_1, N_2 \dots N_n$ (and the parameters contained in them, fixing different concepts) do, in fact, get a certain new interpretation in the light of the new theory (in this case in the light of the theory of relativity). This means that the intensionals of the terms in $N_1, N_2 \dots N_n$ are altered at a certain level of knowledge, which is evidence of a difference between these propositions and the corresponding ones of Newtonian mechanics that took shape before the origin of the theory of relativity. But these symbolic expressions are extensionally the same (identical); they signify one and the same formulas with one and the same physical quantities and meanings, irrespective of their interpretation. The values calculated from the formulas of Newtonian mechanics before development of the theory of relativity, and from the corresponding formulas as limiting cases of propositions $E_1, E_2 \dots E_n$ in the new theory, are identical. The difference of the intensionals of propositions $N_1, N_2 \dots N_n$ does not mean that the identity in their extensionals is purely fortuitous. This identity of the different expresses the sameness, and similarity, and the dialectical relation in essence of the propositions of Newtonian mechanics and of propositions $N_1, N_2 \dots N_n$.

During the revolutionary replacement of one theory by another their dialectical relationship determined by the categories of identity and difference is discovered. The unity of identity and difference is also manifested in the transition from theory T_1 to the generalised theory T_2 through concretisation of the subject-matter by the imposition of certain limitations on it and through reductions of theory T_2 to theory T_1 on the basis of the correspondence principle.

When one is considering the relations between the theory of relativity and classical mechanics from the angle of their adequacy, one must not characterise the first as true and the second as simply applicable in practice, which ensures its acceptability but not its truth. If a theory (or some bit of it) is

applicable in practice that signifies its truth. The point is that the truth of the initial theory (in this case, Newtonian mechanics) is linked with greater 'coarsening' and 'deadening' of the reflected reality, and fewer approximations to it compared with the truth of the generalised theory (in this case, the theory of relativity). The latter is a more exact reproduction of it. Here we meet a situation similar to the following. Say we have measured the length of a fence, which proves to be 500 m 2 mm. In view of our practical needs we can ignore the two millimetres and say that the length of the fence is 500 metres. In both cases we are dealing with true statements. In the second case our statement is still true because we can, if necessary, make our measurements more accurate. It is also important to remember that the accuracy of measurements is always relative. When that is not taken into account, the first statement should be declared false.

§ 6. Generalisation of Objects and Situations by Idealisation

The main forms of idealisation. First, by idealisation is meant a certain mental process of creating *idealised objects*.⁶⁵ Let us consider several examples of the introduction of such objects into science.

Example 1. Suppose we are pushing a cart along a road.⁶⁶ After a shove it rolls along for a time and then stops. There are several ways of increasing the distance it travels after a push; for example, by oiling the wheels, or smoothing the road. The easier the wheels turn and the smoother the road, the longer the cart will move. Oiling of the wheels and smoothing of the road reduce the external influences on the moving body, i.e. the friction. The following dependence can be established by experiment: the smaller the external effects on a moving body the greater will be the distance it will travel. We can invent more and more new ways of reducing the external influences on the body, and correspondingly more and more ways of lengthening its travel, but it is impossible to eliminate all external influences.

The pattern discovered (the dependence between the external influences on a moving body and the length of the path it travels) makes it possible to bring out a certain property

of bodies, which consists in the point that, if all external influences on a moving body are eliminated, it will move infinitely (and, moreover, uniformly and in a straight line) or remain at rest. This property was discovered by Galileo.

Thus, on the basis of a pattern established experimentally, and also relying on some mental experiment, an idealised object, viz., 'inertia', is formed. The term 'inertia' was introduced as an abbreviation for the description of the property of a body to maintain rectilinear, uniform motion continuously (unless it is at rest) in the absence of any external influences on it. As soon as external influences develop, this property ceases to exist.⁶⁷

The shaping of the concept of inertia as a kind of idealised object thus came about through the eliminability of certain conditions that a moving body exists in, and which cannot in principle be eliminated in a real experiment in the absolute sense. This eliminability is assumed in the conditions of a mental experiment.

An idealisation of an absolutely black body was introduced into physics in a similar way. Physical bodies, we know, absorb part of the radiant energy falling on them, reflect part, and let part pass through them. An absolutely black body will be one that absorbs all the radiant energy falling on it, i.e. no part of this radiant energy will be reflected or allowed to pass through it.

Bodies possessing this property do not exist in nature. But physical bodies can be so ordered according to their absorptivity that they are distributed in increasing series from the lowest value of absorptivity (i.e. the lowest value of the relation of the radiant energy absorbed to all the radiation falling on the body) to bodies at the end of the series with an absorptivity approaching but not reaching unity. By employing a mental experiment based on the pattern brought out, we assume the existence of physical bodies with an absorptivity equal to 1, i.e. so-called absolutely black bodies.

Such an idealised object as an absolutely solid body has been introduced into science in the same way.⁶⁸ I could also demonstrate that the concepts of an ideal gas, an ideal solution, a material point, etc., have been introduced into science in the manner described above.

Example 2. Idealised objects have been introduced into science in several other ways. We often identify the shapes of certain natural objects with ideal geometrical ones on the grounds that their deviations from the mathematical shapes are insignificant and inessential for the certain purposes of scientific inquiry. The advantages of such identifications are immense; they make it possible to employ various mathematical apparatuses in their comparatively simple variants in research.

The actual surface of the earth, with its infinite and varied alternations of rises and falls, is very irregular. In order to get a notion of the shape of the earth, we study not its real surface but some theoretical one, introducing into the concept of the shape of the earth an element of abstraction from its existing irregularities, i.e. by looking at it as it were from quite a considerable distance at which these irregularities do not show. Such a method is quite justified since the radius of the earth is very great compared with the highest mountains and the deepest oceanic troughs, and the existence of mountains and troughs does not disrupt the general 'mathematical' shape of the planet.⁶⁹

Example 3. The influence of external effects on an object can sometimes be altered in a different way, and the properties of the object interesting us altered quite insignificantly by the influence of these conditions. In this case a certain idealised object with property *P*, and possessing the property of a certain absolute stability and invariance in relation to the given conditions can be introduced into science.

Idealised objects called incompressible liquids were introduced into hydrodynamics on the basis of those considerations. It is known that the compression of liquids and change in their volume, even under very great pressure, are quite insignificant. On those grounds changes in the volume of liquids arising through the influence of pressure can be abstracted and they can be regarded as incompressible, and as possessing this invariant property in a certain absolute sense. In other words the idealised object of an incompressible liquid is introduced in that way. In contrast to the process of abstraction we ignore here in what the liquids are identical with one another.

Examples 1, 2, and 3 characterise different kinds of idealisation as processes of the forming of classes of idealised objects.

By generalising Example 1, with the formation of an idealisation of inertia, we can formulate a *definition of the first type of idealisation*. It represents a mental experiment built up of the following stages:

(1) by changing certain conditions in which the studied object is found, we make their effect gradually diminishing (sometimes correspondingly increasing);

(2) in that way it is discovered that certain properties of the object studied are also altered uniformly;

(3) by assuming that the effects of the conditions on the object studied have been reduced to zero or have reached a certain invariant factor, we complete the transition to the limiting case and so to a certain idealised object.

This form of idealisation is associated with the supplementing of a certain real experiment by a mental one. The mental experiment in this case consists in constructing an assumption about the reduction to zero of the effect of the conditions on the object studied. Such an experiment is linked with the transition to a certain limiting case.

In types 1 and 2 of idealisation the mental experiment is aimed at realising a certain limiting case unrealisable in reality: the idealised object is explained as existing in reality on the basis of certain disclosed patterns of approximation to the limiting case, and on the basis of practical and theoretical considerations for identifying the ideal model and the real pattern.

Secondly, *by idealisation is meant a mental process consisting in constructing hypotheses in which certain conditions of the existence of the object studied are either eliminated (including abstraction) or modified, which leads to discovery of special idealised properties of the object 'in pure form'.*

This kind of idealisation leads to discovery of the essence of the studied object (situation), and its liberation from attendant factors that obscure or mask the essence of the matter, and provides the possibility of clarifying the essence in a 'pure form', freed from random variables. Such idealised assumptions were widely employed by Karl Marx in *Capital*.⁷⁰

Rubinstein identified the essence of fruitful and creative abstracting activity with idealisation of the second kind.

Abstraction ... consists least of all in a subjective act of a negative

order—a discounting or ignoring of some circumstance or another; it consists in bringing out what is the function of a thing or phenomenon and its dependence on other phenomena when the external circumstances masking or altering them are excluded. The internal properties of a thing are those that function in 'pure form' when the effect of all the masking attendant circumstances in which they are usually perceived are removed.⁷¹

Similar idealised assumptions are widely employed in mathematics and mathematical natural sciences. In them they perform the role of postulates in which some problem that is insoluble in principle in general form in a practical, experimental way, is taken to be solved in a certain absolute sense, on the grounds that it is soluble in an experiment within certain limits and in a certain sense. These assumption-postulates are counted among our constructive possibilities (for example, in mathematics, where we constantly have to solve mass problems in designing; in physics, in which mass problems are formulated for experimental-measuring procedures). The process of idealisation here (and, incidentally, in its other forms as well) is associated with conversion of the relative in experience into an absolute in theory.

This can be seen from the following examples.

A certain problem, for example the joining of two points in space by a straight line (by means of the appropriate instruments) is not always soluble in experimental work, but Euclid assumed it to be soluble in theory in general form and included it among the postulates of the theory.

In mechanics there is a postulate that we can measure the velocity of a moving body at any point on its path, yet that cannot be done either in relation to all the points of the path or in relation to any separate point, since there are no points without extension in nature.

Generalisation and idealisation. We can derive generalisations from the idealised objects formed; for that, the idealised objects are regarded as idealised properties. The idealised 'object' 'inertia', for instance, is a property of a body that continuously maintains uniform motion in a straight line (if it is not at rest) in the absence of any external influences on it. An idealisation like a black body is also the property

of a body to absorb all the radiant energy incident upon it. Furthermore, the terms 'inertia' and 'black body' can be looked upon as abbreviations for describing the corresponding idealised properties. Similarly an incompressible liquid is the property of a liquid not to change its volume when different pressures are exerted on it.

The idealised objects examined above can thus be represented as the corresponding idealised properties by which real objects are understood when regarded in the conditions of applying suitable mental experiments to them. This permits classes of idealised objects to be formed, e.g. classes of inertial systems, black bodies, incompressible liquids, and spherical bodies. As a result of generalisation, consequently, based on abstraction by identification, not only are classes of real objects introduced into science (as are also classes of abstract objects, for example, prime numbers), but also classes of idealised objects.

The programme that Euclid outlined in his *Elements* assumed the finding of algorithms for solving various classes of problems in construction by means of compasses and rulers. These problems included one of finding general methods allowing any section of a straight line to be bisected, a tangent to a circle to be drawn from any point outside it, and so on. To simplify realisation of this programme Euclid introduced certain idealised assumptions.

It is assumed, for example, that we are dealing with a bit of a plane that can always be increased at will (as Euclid himself did) rather than some given piece of surface; that a point has no measurement in general, that the compass and ruler are ideal ones by which any two points in space can be joined by a straight line, or a circle can be described with as large or as small a radius as is convenient.⁷²

The algorithm for solving a certain mass problem in Euclidean geometry thus becomes one of reducibility; the answer to what it means to solve a problem must be that it means to reduce its solution to problems that are taken as solved in our idealised assumptions.

Why is the adoption of these assumptions linked with the process of generalisation? The point is that if they were not adopted, a geometry of its own would have to be developed for every given compass and ruler, and for every given bit of sur-

face, and special rules formulated to solve the problems. The deeper mathematical essence, of a general character, of the problem being solved is brought out through assumptions of this kind.

By modelling real situations by adopting certain idealised assumptions we can then apply the created idealised model to other real situations insofar as they are regarded within the context of these same idealised assumptions. This means that the initially created idealised model of some real situation (or certain real situations) can subsequently be generalised and transferred to other cases. Then we shall be dealing with the generalisation of an idealised model.

§ 7. Statistical and 'Approximate' Generalisations

A doctrine of *collective concepts* was developed in traditional logic. To them correspond individual concepts consisting of several uniform units linked into a single whole, sometimes by quite complex relations characterising their structure, functions, and other aspects. In contrast to ordinary sets, these objects are often called *aggregates*, *systemic objects*, and *systems*.

The constellation Ursa Major, for instance, is not simply a set of star-elements, but a certain aggregate with a special structure and position among the other constellations. This is manifested in its being possible, in relation to this aggregate, to formulate a statement, for example that 'the constellation Great Bear has the shape of a dipper', in which the predicate 'has the shape of a dipper' refers to the whole set of stars constituting the aggregate, and not to each element of it taken separately. (Compare, also, the statement 'this family has a fortune of a million'.)

Ordinary sets reveal the 'wholeness' of their formation, which consists of many, though identical elements, when we express predicates about them that characterise their magnitude, the relations between magnitudes, and their structural features. These predicates refer to the sets as whole aggregates. The following statements are of this kind: 'The set of people is quite numerous', 'set M is countable', 'set M is numerically equal to set N ', 'the given set is standard'.

We have the set as a whole in mind, both when we are

discussing its qualitative composition, and in particular when we say that 'a considerable part of the population of the city consists of students'. When characterising the percentage composition of some set we also characterise the set (aggregate) as a whole. We can state, for instance, that 40 per cent of the population of a collective farm consists of males, having calculated that 96 of the 240 persons living in the farm are males. The 40 per cent refers to the population of the farm as a whole, and not to each member of it, who is either a female or a male. In the cases considered we are not dealing with a process of generalisation; having studied a set or aggregate from the angle of value characteristics, we then state them relative to the aggregates or sets viewed as a certain, individual whole.

More interesting cases arise when we begin to examine certain *statistical processes*, based on the theory of probability, whose field of application extends to mass events (phenomena) to which the *law of large numbers* can be applied.⁷³

The sense of this law is that the mean of independent uniform quantities calculated *a priori* will differ as little as desired from the mean obtained *a posteriori* (by way of mass observations), when the condition of the scrutiny is a quite large set of independent quantities. In other words this is a very general law in accordance with which the aggregate action of a large number of chance factors inevitably leads in certain conditions to a result that is almost independent of chance.

The *classical definition of probability* was formulated as the relation of a number of favourable cases to all possible ones. For that it was assumed that all the cases examined were equiprobable and equipossible.⁷⁴ This equiprobability of the examined events (and at the same time their chance character independent of one another), is disclosed by means of the so-called *principle of indifference*, according to which there are no grounds for preferring the happening of one event (outcome of an experiment) to that of another and for deciding why one event should happen more often than another. If we take an ideally fashioned die, we have no grounds for preferring one side to come up when it is thrown than another; furthermore we have grounds for considering it equiprobable that any one of the sides will turn up. The probability of one of two sides (for example, the ace or the six) coming up will be equal

to the relation of the number of cases favourable for this situation to all possible cases, i.e. $\frac{2}{6} = \frac{1}{3}$. The classical theory of probability dealt, for example, with cases when the probability of such and such an event, calculated on an *a priori* basis, is confirmed *a posteriori*, i.e. by an experimental check.

In frequency theory, which is due to von Mises and Reichenbach, the concept of *relative frequency* is the key one. This is the relation of the number of appearances of the studied event in a series of tests to the total number in which this event could appear. This theory makes it possible to tackle a problem the opposite of the classical one, viz., to judge the probability of the mass random events being studied from the results of their relative frequency. For that, however, there must be many tests.

The application of mathematics to study of phenomena of this sort is based on the point that in many cases, when one and the same experiment is repeated many times in the same conditions, the *frequency of the result being examined* (i.e. the ratio of the number of experiments in which this result is observed to the total number of experiments made) *remains roughly identical all the time*, and close to a certain constant figure p .⁷⁵

The frequency of throwing a six with a gaming die, for instance, will be $1/6$ with a large enough series of throws; the frequency for an arrow's hitting the target in one and the same conditions with a considerable number of shots will remain almost the same. The percentage of spoiled articles in any series of tests in one and the same production unit, in identical conditions will be roughly the same. Other conditions being equal, the relative frequency of the birth of boys in a certain region (with long observation) will not deviate in any way significantly from the constant p , which represents the probability of this event. Only with a large number of tests do the constant ratios recorded in statistical laws become clear; such as, for example, the laws of inheritance discovered by Mendel.

The occurrence of separate individual events can be calculated from the frequency (statistical) theory.⁷⁶ Special statistical methods of analysis have been developed from it that have found wide application in particular in the social sciences.

Let us consider two types of generalisation:

(1) *statistical*, based on the frequency theory, and patently employing statistical methods of analysis;

(2) *approximate*, based on observations that do not presuppose the application of such methods of investigation in patent form, but which are also statistical by nature since these methods are tacitly employed.

Let us begin analysis of the first type of generalisation (1) from an example. Assume that we want to know what percentage of men living in a big town with a population of several hundred thousand use an electric razor. For that let us take a sample of 1,000 men (in accordance with the methods developed in statistics) and question them. Assume that 800 of them answer that they shave with an electric razor. The relative frequency of the studied feature will be 0.8 (it is a chance circumstance for these objects that a man may shave with an electric razor or with another kind, say a safety razor). This *frequency* of the studied feature in the sample is a *generalisation* of the results of an experimental-statistical survey of individuals. *In this case the people studied in the sample are generalised from the angle of a certain feature P into a concept of the relative frequency of men who shave with an electric razor.* It turns out that our sample as a whole ('collective') is characterised by most of its members (i.e. 80 per cent) having the property of 'shaving with an electric razor'.

By employing statistical methods we have formulated a concept of a certain aggregate. In it we are dealing with a generalisation of the individuals of the sample such as leads once again to a concept of the whole individual aggregate (sample). This will be the *first step* in our generalisation. But this concept ('initial generalisation') about the aggregate can also be interpreted as a set distinguished by the following common property: it is true for each of its elements that the probability, when some one of them is chosen, is 0.8 that it will be a man who shaves with an electric razor. This common feature of the elements of the set formed is evidence that it was formed as a result of generalisation of the separate individuals of the sample.

The *second step* in generalisation (1) is *to pass from the sample to the population*; we conclude that the statistical probability in the population of the feature 'a man who shaves with

an electric razor' (P) will be 0.8, the same as in the sample. This transition is *a transfer of the predicate from the sample to the broader aggregate (group), the population*. This aggregate, i.e. the population, can be regarded in relation to the feature as a certain set distinguished by the following property, viz., it is true for each element of it that the probability of coming across a man who shaves with an electric razor is 0.8 when any one of them is taken. We thus transfer the recorded predicate P from the smaller set to a larger one. At this stage of generalisation (1) we are dealing with *generalisation of the judgment*. Strictly speaking it is a broadening of the concept (see Chap. II, § 1).

The relative frequency in the sample does not coincide in the general case with the frequency in the population. The point is that a certain probability of relative frequency ('statistical law') is revealed quite exactly only with a very large number of tests. In our sample, which (as we suggested) was made according to all the rules of statistical investigation, we are dealing with a limited number of tests.

The first type of statistical generalisation thus consists of two stages:

(1) a synthetic generalisation of the individuals studied in the sample from the angle of the relative frequency of some feature P ;

(2) transfer of the relative frequency of feature P in the sample to the whole population. This transfer is a generalisation of a certain statement (judgment) about the relative frequency of property P in the sample to the statement about the relative frequency of this property in the population. We thus treat the sample and the population as sets rather than aggregates.

From statistical generalisations we can draw deductive probability conclusions.⁷⁷ For example,

(1) the relative frequency of men who shave with an electric razor in town N is 0.8;

(2) here is a man from town N ;

(3) the probability that he shaves with an electric razor is 0.8;

This is usually written as follows:

(1) $rf(Q, P) = 0.8$;

(2) Pa ;

(3) Qa.

Carnap suggested interpreting the deduced conclusion Qa as follows: on the basis of (1) and (2) statement (3) has a probability of 0.8.⁷⁸ In Carnap's interpretation this conclusion is thus based on a dual probability, viz., statistical (in the first premise) and logical (in the conclusion), in which an evaluation is made not of the probability of the events but of the hypothesis.

Inferences of that kind—from statistical laws to a partial case—underlie the corresponding predictions of future separate events. If we know, for instance, that the relative frequency of encountering a blond in our area is 0.7, then (if the distribution of blonds is not subject to any law) it can be inferred that: 'the probability that the next person I meet in our area will be blond is 0.7'. In essence we also have employed a *deductive probability conclusion* above, interpreting the statement about the aggregate (group) as a statement about a set.

The *second type of generalisation* differs from the first in not explicitly employing statistical methods of investigation; in it the generalisation is the result of conditional observations, and of long human experience, 'roughly' weighing up many observations. Following Chelpanov, I shall call such generalisations *approximate*.⁷⁹ They are expressed in ordinary language as judgments with the following form: 'Most *S* are *P*'. In judgments expressing approximate generalisations, such words and word combinations as 'very often', 'most often', 'for the greater part', 'ordinarily', 'usually', 'in general', and so on, are employed instead of 'most'. Examples are the following: 'most women want to have children', 'prolonged hypertension very often leads to cardio-vascular ailments', 'severe trials usually steel people'.

When we want to stress that our approximate generalisations differ little from general statements we often employ expressions like 'as a rule', and 'the overwhelming majority', instead of 'the majority' or 'most'. For example, 'as a rule people protect life', 'the overwhelming majority of people strive for peace'. Sometimes the word 'majority' and ones close to it in meaning are not expressed explicitly but only implied, as for example 'Georgians are hospitable' (implying that 'the majority of Georgians are hospitable' or even 'the overwhelm-

ing majority of Georgians are hospitable'). Despite the fact that approximate generalisations of this kind come about spontaneously, as a result of conditional generalisation of people's long experience, they can be regarded in a certain interpretation as intuitively made statistical generalisations; it can be supposed that the initially generalised experience is experience of a certain sample that is later extended to a broader domain (including the future).

For each concrete judgment of the form 'most S are P ', the corresponding concrete judgments of the form 'the given S is not P ' are obviously not contradictory. An approximate generalisation may also have the form of a judgment 'most S are not P ' (compare 'most men are not bachelors').

Statistical generalisations of the first type (like any probability-statistical investigations in general) are justified since in the appropriate theoretical systems (in the mathematical theory of probability and in statistics, based on this theory) a postulate is employed that chance variables have a tendency to offset one another. This postulate is legitimate since it finds confirmation in experience. But it has to be explained at the *philosophical, theoretical level*. In any case it is only clear that the objects investigated in the theory of probability and statistics are subject to the effect not only of variable, changeable, chance factors operating at random, but also of certain stable, constantly influencing causes, whose cumulative effect on the objects studied comes out with a large number of tests. These are seemingly certain stable factors of a systemic character: the regularities connected with one side of a die or another coming up depend not only on chance factors connected with the act of throwing but also with the rigid, invariant, systemic properties of dice. In this there is the complex dialectic of necessity and chance.

¹ The complex dialectic of the particular and the general that I have noted, associated with this 'merging', is the basis for identification of properties in the theory of sets (in several contexts), and of the sets corresponding to them.

² On this point see D.P. Gorsky. *Voprosy abstraktsii i obrazovanie ponyatii* (Problems of Abstraction and the Formation of Concepts),

- Izd-vo AN SSSR, Moscow, 1961, pp 131-137.
- ³ Maxim Gorky. *The Lower Depths*. Translated by Laurence Irving. (T. Fisher Unwin, London, 1912), p 179.
- ⁴ See M.M. Novoselov. 'Identity'. In *Filosofskaya entsiklopediya*, Vol. 5 (Sovetskaya entsiklopediya, Moscow, 1970).
- ⁵ See T.D. Gorskaya. Identity on the synchronic plane and its types. *Filosofskie nauki*, 1978, 5: 77.
- ⁶ See A.I. Uemov. On the principle of identity. *Voprosy filosofii*, 1969, 6: 87.
- ⁷ See T.D. Gorskaya. *Art. cit.*, p 77.
- ⁸ Judges' different evaluations of the performances of one and the same sportsmen (for example in gymnastics, figure skating, etc.) are evidence of the complexity of developing single criteria of assessment that would enable different sportsmen to be distributed by different classes (in the sense of qualifications) and to include identical sportsmen (in the sense of their qualifications) in one and the same class.
- ⁹ For the symbolic form of notations see Chapter II, § 1.
- ¹⁰ The formation of initial classes of generalisation includes a certain essential analytical moment: we agree to consider only those chemical elements to be metals that possess properties which are reduced to the complex property of being a metal. If chemical elements were discovered which did not possess even one of the properties forming part of the complex property of being a metal, we should not consider them to be metals. That condition is quite 'rigid'. We reject it when the need matures for a general reconstruction of the established typology (on that, see Chap. III, § 3).
- ¹¹ Our evidence is summed up in supposition 7): a set is indicated from which the subjects of the statements that represent our evidence are derived.
- ¹² V.N. Kostyuk. *Metodologiya nauchnogo issledovaniya (Methodology of Scientific Research)*, Vishcha shkola, Kiev-Odessa, 1976, pp 59-60.
- ¹³ See Chapter II, § 5.
- ¹⁴ See D.P. Gorsky. The anticipatory character of reflection of reality at the level of human cognition. In *Praktika i poznanie*, Nauka, Moscow, 1973, pp 54-59.
- ¹⁵ See A.L. Subbotin. The principles of Locke's epistemology. *Voprosy filosofii*, 1955, 2: 105-107.
- ¹⁶ John Locke. *An Essay Concerning Human Understanding* (George Routledge & Sons, London, 1944), p 329.
- ¹⁷ A.N. Leontiev. Thinking. *Filosofskaya entsiklopediya*, Vol. 3 (Sovetskaya Entsiklopediya, Moscow, 1964), p 518.
- ¹⁸ Cornel Popa. *Teoria definitiei* (Editura stiintifică, Bucharest, 1972), p 104 (translated from the Russian).
- ¹⁹ It is expedient here, following Gottlob Frege, to differentiate between *Sinn* and *Bedeutung*, the reference and the sense of a term. Cornel Popa does not make this differentiation, having in mind that the meaning of a term is a unity of its extensional (in this case, the initial class of generalisation) and intensional (its corresponding elementary concept).
- ²⁰ On this see D.P. Gorsky. *Definition* (Progress Publishers, Moscow, 1981), pp 79-85.
- ²¹ Christoph von Sigwart. *Logik* (Freiburg, 1889), p 322.

- ²² Ernst Cassirer. *Substanzbegriff und Funktionsbegriff*, 2nd ed. (Verlag von B. Cassirer, Berlin, 1923), p 8.
- ²³ *Ibid.*, p 19.
- ²⁴ *Ibid.*, p 33.
- ²⁵ N. Lange. *Uchebnik logiki* (Textbook of Logic), Odessa, 1898, pp 36-37.
- ²⁶ Edmund Husserl. *Logische Untersuchungen* (Max Niemeyer, Halle, 1901), p 166.
- ²⁷ See S.A. Yanovskaya. *Metodologicheskie problemy nauki* (Methodological Problems of Science), Mysl, Moscow, 1972, pp 34-75.
- ²⁸ See *Entsiklopediya elementarnoi matematiki* (Encyclopedia of Elementary Mathematics), Vol. 1 (Sovetskaya entsiklopediya, Moscow, 1951), p 15.
- ²⁹ V.I. Lenin. *Materialism and Empirio-Criticism* (Progress Publishers, Moscow, 1977), p 126.
- ³⁰ *Ibid.*, p 88.
- ³¹ Karl Marx. Randglossen zu Adolph Wagners "Lehrbuch der politischen Oekonomie". In: Marx/Engels, *Werke*, Vol. 19 (Dietz Verlag, Berlin, 1962), p 363.
- ³² P.Y. Halperin. *Vvedenie v psikhologiyu* (Introduction to Psychology), Izd-vo MGU, Moscow, 1976, p 112.
- ³³ See Constance Reid. *Hilbert* (George Allen & Unwin, London, 1970), p 104.
- ³⁴ V.V. Davydov. *Vidy obobshcheniya v obuchenii* (Forms of Generalization in Teaching), Pedagogika, Moscow, 1972, p 326.
- ³⁵ See S.A. Yanovskaya. *Op. cit.*, pp 34-75
- ³⁶ The individual variables x, y, z in these axioms are linked by universal quantifiers. The variable for the equality-type relation P is free. In the substitution described above the quantifiers are eliminated through simultaneous consecutive substitution of individual elements of the domain for x, y, z .
- ³⁷ Karl Marx. *Capital*, Vol. 1. Translated by Samuel Moore and Edward Aveling (Progress Publishers, Moscow, 1978), p 19.
- ³⁸ See *Shornik statei po filosofii matematiki* (Collected Articles on the Philosophy of Mathematics), Moscow, 1936.
- ³⁹ These questions also occupied Federigo Enriques in connection with problems of the forming and definition of concepts (see Federigo Enriques. *Storia del pensiero scientifico*, Vol. 1 (Milan, 1932).
- ⁴⁰ See Y.A. Shreider. *Ravenstvo, skhodstvo, poryadok* (Equality, Similarity, and Order), Nauka, Moscow, 1971, p 50.
- ⁴¹ The application of definitions by abstraction to empirical knowledge outside mathematics (analysis of the concept of value is such) is associated with a number of difficulties. The point is that the property of the transitiveness of relation R is not realised in the general case. Thus, in the exchange of commodity a for commodity b , and commodity b for commodity c , commodity a will not necessarily be exchangeable for commodity c , since the process of exchange takes place in time, and goods may lose their commodity quality and become useless. Even more examples of this kind can be adduced when sensually perceptible properties are being distinguished. That means, strictly speaking, that relation R is not an equality-type relation in the cases considered but a relation of *sameness*

- or *similarity*. But I shall retain the theory of definition by abstraction for empirical knowledge by assuming several idealisations, and first and foremost by assuming the feasibility of the property of transitiveness. Assumption of that idealisation can be justified by pragmatic considerations.
- ⁴² Karl Marx. *Capital*, Vol. 1, p 59.
- ⁴³ Thomas S. Kuhn. *The Structure of Scientific Revolutions*, Vol. 2, No. 2 (U. of Chicago Press, Chicago, 1970), p 53.
- ⁴⁴ *Ibid.*, p 54.
- ⁴⁵ The operational-genetic standard is formed by operational-genetic definition. By genetic definition is meant definition of an object by description of the specific ways it originates, is manifested, prepared, and obtained. An operational definition (in the narrow Bridgman sense of the term) is realised through description of the specific measuring operations employed for physical magnitudes.
- ⁴⁶ The concept of a fundamental theoretical scheme was introduced into the methodology of science by V.S. Stepin. It is a model of theoretical knowledge that includes abstract objects and the laws corresponding to them in mathematical form, and also the rules of correspondence that link the theoretical laws with the reality being directly studied. Examples of a fundamental theoretical scheme are Newton's laws of mechanics. Scientific pictures of the world can be made in accordance with these schemes. If, for example, a fundamental theoretical scheme (Newton's mechanics) describes natural processes by material points that move in inertial systems, forces, etc., the corresponding picture of the world will be described in terms of corpuscles, and their movement in contexts of absolute time and space.
- ⁴⁷ Richard P. Feynman, Robert B. Leighton, Matthew Sands. *The Feynman Lectures of Physics*, Vol. 1, Part 1, (Addison-Wesley Publishing Co., Reading, Mass., 1977), pp 2-3.
- ⁴⁸ *Ibid.*
- ⁴⁹ V.S. Stepin. *Stanovlenie nauchnoi teorii* (The Construction of a Scientific Theory), Izd-vo BGU, Minsk, 1976, p 143.
- ⁵⁰ V.I. Lenin. *Materialism and Empirio-Criticism*, p 269.
- ⁵¹ L.S. Stebbing. *A Modern Introduction to Logic*, 2nd ed., 1933. Cited from Richard Robinson. *Definition* (Clarendon Press, Oxford, 1950), p 141.
- ⁵² See D.P. Gorsky. *Definition*, pp 100-102.
- ⁵³ Frederick Engels. *Dialectics of Nature* (Progress Publishers, Moscow, 1974), p 224.
- ⁵⁴ *Ibid.*
- ⁵⁵ *Ibid.*, p 225.
- ⁵⁶ *Ibid.*
- ⁵⁷ See D.P. Gorsky. A scientific theory and the ways of generalising it. *Voprosy filosofii*, 1966, 8: 45-54.
- ⁵⁸ See I.V. Kuznetsov. *Printsip sootvetstviya v sovremennoi fizike i ego filosofskoe znachenie* (The Principle of Correspondence in Modern Physics and Its Philosophical Meaning), Gostekhizdat, Moscow, 1948.
- ⁵⁹ *Ibid.*, pp 54-55.
- ⁶⁰ S.V. Illarionov. The principle of limitations in physics and its connection with the correspondence principle. *Voprosy filosofii*, 1964, 3: 100.

- ⁶¹ An example like the following is often cited for that purpose: it is known that when Galileo formulated the law of free fall of bodies *in vacuo*, he went against the observable facts, which refuted the law established by him. In this case these facts (the 'language of observations') were interpreted, in the light of the conception he had developed, in such a way that they ceased to contradict it.
- ⁶² Thomas S. Kuhn. *Op. cit.*, p 101.
- ⁶³ *Ibid.*, p 102.
- ⁶⁴ *Ibid.*
- ⁶⁵ I first considered the problem of the forms and definition of idealisation in my book *Voprosy abstraktsii i obrazovanie ponyatii* (Problems of Abstraction and the Formation of Concepts), pp 276-290.
- ⁶⁶ The example is taken from Albert Einstein and Leopold Infeld. *The Evolution of Physics* (Simon & Schuster, New York, 1942), pp 7, 8.
- ⁶⁷ When discussing the introduction of the idealisation 'inertia' into science, I did not examine several points whose clarification was essential for discovery of the law of inertia (for example, I did not analyse the curvilinear motion always associated with the existence of acceleration, i.e. change of velocity at each point of the path, etc.). But leaving aside these matters and others, which have a bearing on the shaping of a given idealisation, does not disturb the general mode of introducing the new idealised object, inertia, described above.
- ⁶⁸ See G.S. Landsberg (Ed.). *Elementarnyi uchebnik fiziki* (Elementary Textbook of Physics), Vol. I, Prosveshcheniye, Moscow, 1956, pp 119-120.
- ⁶⁹ S.V. Kalesnik. *Osnovy obshchego zemlevladieniya* (The Principles of Common Land Ownership), Uchpedgiz, Moscow, 1955, p 20.
- ⁷⁰ On this point see Chapter II, § 3.
- ⁷¹ S.L. Rubinstein. *Bytie i soznanie* (Being and Consciousness), Izd-vo AN SSSR, Moscow, 1957, pp 107-108.
- ⁷² S.A. Yanovskaya. *Op. cit.*, p 190.
- ⁷³ Here I shall examine the processes of generalisation connected only with statistical (classical and frequency) probability, and shall not employ the logical concept of probability.
- ⁷⁴ The understanding of probability in the classical theory was criticised by supporters of the frequency theory (in particular by Richard von Mises and Hans Reichenbach). They claimed that the definition of probability by the ratio of equiprobable favourable and possible cases contained a vicious circle. In my view this criticism has no basis. The point is that one cannot identify probability with equiprobability. Probability is the property of events, while equiprobability is the relation between them, and a relation moreover of an equality type (i.e. a relation that is symmetrical, transitive, and reflexive). This relation brings out and reveals the property of their probability in random events (see Chapter I, § 3). But the spokesmen of the frequency theory were right in their critique of the classical theory in stressing its narrowness and its applicability only to simple, and not very interesting cases in which the equiprobability of the outcomes of experiments is easily substantiated by means of *a priori* considerations (for example, the fall of one of the six sides of a thrown die, the cutting of one card or another from a well-shuffled pack, and so on). When the problem

is the application of the concept of probability in the social sciences, certain branches of physics, life insurance, weather forecasting, etc., the problems of understanding, substantiating, and calculating probabilities are more complicated.

⁷⁵ A.M. Yaglom, L.M. Yaglom. *Veroyatnost' i informatsiya* (Probability and Information), Nauka, Moscow, 1960, pp 13-14.

⁷⁶ See Rudolf Carnap. *Philosophical Foundations of Physics* (Basic Books, New York, 1966), pp 27-28.

⁷⁷ Carnap called them inductive conclusions in the general case, and the forms that we are considering here 'predictive inferences'. (See Rudolf Carnap. *Op. cit.* p 39).

⁷⁸ R. Carnap. *Op. cit.*, p 37.

⁷⁹ See G.I. Chelpanov. *Uchebnik logiki* (Textbook of Logic), Gospolitizdat, Moscow, 1946, pp 121-122.

Chapter II

ANALYTIC MODES OF GENERALISATION

§ 1. The Logico-Analytic Generalisation of Concepts. Extension and Localisation of Concepts. Deepening (Concretisation) of Concepts

In textbooks of formal logic the process of generalisation of concepts is defined, following the ancient philosopher Porphyry as one of transition from an initial concept to one resulting (generalised) through abstraction of all the specific attributes (characteristics) of the initial concept and leaving in its content only those of its attributes that belong to each of the elements of the class (range) corresponding to the initial concept, but not only to those elements of the class. This means that the class of objects of the concept is broadened during the generalisation and includes that of the initial concept as a normal part of it. By generalising the concept of a square, we can thus arrive at the concept of a tetragon. For that we abstract from the specific attributes inherent in any square, and only in a square, which include that for example, of 'having equal sides and equal angles'. Those attributes, inherent in all squares but not only in them (e.g. of 'being a quadrangular figure'), are preserved in the content of the generalised concept of a tetragon.

By comparing the class of objects of the initial and the generalised concept we find that the range of the first, initial concept ('square') forms part of that of the second ('tetragon') and constitutes a normal part of it. So

the generalisation of a concept is a logical operation that makes it possible to pass from concepts of a smaller class of objects to ones of a larger class by discarding attributes belonging only to the objects that form part of the concept being generalised.¹

Voishvillo gives a similar definition, though he writes of species differences instead of the concept of discarding attributes

inherent only in the objects that form part of the concept being generalised.

The *operation of generalisation of a concept* is defined in traditional logic as the transition from a certain concept to another one broader in class of objects by excluding from the content of the initial concept any of the attributes that constitute the species difference of the objects generalised in it.²

In all definitions of generalisation there is the implication that the content of the generalised concept constitutes a normal part of the concept generalised. When, as a result of transformation of the initial concept, we get a resulting concept of the same class of objects as the initial one, we will not be dealing in the general case with a generalisation but only with a certain equivalent transformation of the initial concept. We can get a concept 'tetragon', for instance, from the concept 'square' through generalisation of the concept, or the concept 'writer' from the concept 'dramatist', 'vertebrate' from 'mammal', 'triangle' from 'equilateral triangle'.

By starting from a generalised concept we can deduce the previous initial concept from it, for which it is sufficient to add to the attributes of the generalised (and now already initial) concept those that relate only to the normal part of the class of objects (set) of the initial concept. This process is the technique of logical *limitation of the concept*. The passage from the concept of a tetragon to that of a square, for example, is such a process of limitation or restriction. If, during generalisation of a concept, the generalised concept relates to the initial one as *genus to species*, then during restriction of the concept, the limited one relates to the initial concept as *species to genus*. By starting from a certain initial concept, we can subsequently obtain ever broader generalisations through analytical generalisation.

The concept 'object of thought' will be widest for any concept generalised, since an object corresponding to any content of the concept can always be the subject-matter (object) of study. When the operation of generalising a concept is regarded as one of generalising its class of objects, then, at the level of generalisation preceding the concluding one, ('object of thought'), all objects can be classed as a category or thing (set of things), or property, or relation, if the concept of the thing covers what

we call events, states, actions, etc. During the generalisation of scientific theories (at least of empirical theories) we employ synthetic generalisations (although they may also include individual moments associated with analytical approaches). Examples of such generalisations were examined above (Chap. I, §§ 4 and 5). The limits of the generalisation of scientific theories are an extremely complex problem and call for the application of concrete-historical and dialectical approaches.

Voishvillo has investigated the process of analytical generalisation of concepts;³ I regard his work as innovatory. In his approach to generalisation he employed the theory of concepts he had developed, and applied the process of logical deduction to forms expressing the content of the concept. He defined concept as

*an idea that is the result of generalisation (and distinguishing) of objects or phenomena of one class or another according to more or less essential attributes (and therefore also common to these objects and in the aggregate specific form, distinguishing them from the multitude of other objects and phenomena)*⁴.

The concept as an idea of objects of reality defined through their properties is written in the form of the formula $xA(x)$; 'those x that have the property A '. By A , in the case of attributive concepts, is meant a certain property or conjunction of properties. The symbol A usually signifies the main content of the concept, and by its content is generally meant the whole aggregate of attributes, including those logically deducible from the main ones (as a result of employing the laws of logic and the proven statements of the theory to which the concept belongs).

'The class of objects of the concept $xA(x)$ ', Voishvillo writes, 'is the set $WxA(x)$ (which apparently represents the set of truth of the common name expressing this concept)'.⁵

Take the main content of the concept 'to be a rectangle and have equal sides'. The property $A(x)$ can be written in the form of a compound predicate: 'The rectangle $(x) \wedge$ has equal sides (x) '. What is x here? Obviously it is a variable whose domain of possible meanings is a class of plane geometrical figures. As a whole the concept of plane geometrical figures, rectangular with equal sides, can be written as ' x (rectangle $(x) \wedge$ has equal sides (x))'. That is a concrete expression for

the concept $x A (x)$, in which $A (x)$ is the predicate expressing the specific distinction, while the variable x covers plane geometrical figures.

The class of objects of the concept $x A (x)$ can be written as $W x A (x)$ ('that class of plane geometrical figures that are rectangular and have equal sides'). In this case the class (set) of squares obviously constitutes the class of objects of the concept $x A (x)$.

When we have a concept with a two-member predicate, it can be written as $(x, y) A (x, y)$, in which A is a two-member relation: for example, ' (x, y) is greater than (x, y) '. The class of objects of this concept will include the ordered pairs of numbers that satisfy the function ' x is greater than y '.

In the general case the structure of a concept can be written in the form $(x_1 \dots x_m) A (x_1 \dots x_m)$. The class of objects of such a concept $W (x_1 \dots x_m) A (x_1 \dots x_m)$ consists of the ordered sets of objects from the domains of meanings of the variables $x_1 \dots x_m$.

Let us now pass to analysis of the process of generalisation usually elucidated in handbooks of formal logic, in the light of the ideas formulated above. By taking a content-wise definition of the operation of generalisation (remembering, moreover, that the content of the generalised concept constitutes a normal part of the concept generalised), and the general definition of logical movement in the relevant sense (L follows from G , if and only if the content of L is part of the content of G), the operation of generalising the concept can be written symbolically in very general form.

Let the initial concept being generalised have the form $x (A (x) \wedge B (x))$ (1), in which x is a variable covering the domain of its possible meanings, $A (x)$ is a certain aggregate of properties not specifically distinctive of concept (1), and $B (x)$ is the aggregate of specific distinctions. For the concept these distinctions can be regarded as a single specific distinction when they are thought of, for that purpose, as united by a sign of conjunction.

When the corollaries are deduced from the concepts, the variable x standing before the propositional function is eliminated. We can obtain the expression $A (x) \wedge B (x)$, for instance, from concept (1). $A (x)$, moreover, is deducible from

$A(x) \wedge B(x)$, i.e. $A(x) \wedge B(x) \vdash A(x)$ takes place. That in turn signifies that $A(x) \wedge B(x) \supset A(x)$ is logically true. The concept $xA(x)$ will also be generalised in relation to the initial concept $x(A(x) \wedge B(x))$.

Take an example. Suppose we are given the concept 'dramatist'. We write it in the symbolic form $x(A(x) \wedge B(x))$ (1), in which x is a variable whose domain of possible meanings is the sphere of people; $A(x)$ is the property of 'being a writer', and $B(x)$ is the property of 'writing plays'. We can obtain the concept $xA(x)$ from concept (1), i.e. the concept 'writer' from the concept 'dramatist', by abstracting the property of 'writing plays' proper only to certain writers. Since the deduction $A(x) \wedge B(x) \vdash A(x)$ is made, $A(x)$ (i.e. the concept 'writer') is a generalisation of concept (1) (i.e. of the concept 'dramatist').

Voishvillo, characterising the process of generalisation as a whole, wrote: 'this form of generalisation of a concept is obviously a transition from a concept of the $x(A(x) \wedge B(x))$ type to the concept $xA(x)$.'⁶ It is not quite clear here whether the cited formulation is a general formal definition of the generalisation of a concept or is a certain formal characterisation of a content definition of generalisation.

In fact the cited deduction from $A(x) \wedge B(x)$ of $A(x)$ cannot be a formal definition for obtaining the generalised concept $xA(x)$ from the initial concept $x(A(x) \wedge B(x))$, since $B(x)$ as well as $A(x)$ can be deduced from $A(x) \wedge B(x)$. But the concept $xB(x)$ is not a generalisation in regard to the concept $x(A(x) \wedge B(x))$. In our example $B(x)$ is a specific distinction for the concept 'dramatist' being generalised. The concept $xB(x)$ will therefore not broaden (and consequently also generalise) the concept $x(A(x) \wedge B(x))$.

For the formal procedure determined by the formal definition to reflect the essence of the matter, we must know, at the content level, that $A(x)$ in the formula $x(A(x) \wedge B(x))$ is an aggregate of properties belonging to each element of the set being generalised (but not only to it), while $B(x)$ is a specific property for elements of the set being generalised. We can only get this knowledge, however, through a *content analysis* of the concept being generalised. The formula adduced

must, therefore, apparently be regarded simply as an important formal characteristic of the general content definition of generalisation. Such a mode of expression as the following will therefore be more valid: if concept $x A(x)$ is the generalisation of concept $x (A(x) \wedge B(x))$, then $A(x) \wedge B(x) \vdash \vdash A(x)$ will be deducible, which already has a purely logical character.

The symbolic writing of the general concept of generalisation via a corresponding deducibility is thus not a full formalisation of the process of generalisation.⁷ But it can be regarded as such if condition is introduced into the theory that each concept is a generalisation of itself, i.e. if certain 'degenerate' cases are introduced.

We often encounter a similar situation in the neopositivist logic of science, in which the task was posed of defining the concept of a scientific law, a counterfactual judgment, explanation, etc., by purely formal means. These aims, however, were not realised.

The failure of the neopositivist logic of science was due (1) to its mistaken philosophical precepts connected with making an absolute of the apparatus of modern formal logic and exaggeration of its possibilities, and (2) to the inadequacy of the apparatus of modern formal logic (at least the apparatus of classical logic with material implications adapted for the needs of mathematics) for solving the quite complex problems of the methodology of natural science, and the methodology of science in general, and in particular for full formalising of the phenomena mentioned above.

The question arises whether it is possible in general, in such a case, to employ the apparatus of modern formal, (i.e. mathematical) logic to analyse the methodological problems of experimental knowledge. Yes, it is possible.⁸ But formal logic can be successfully employed to analyse complex methodological problems of experimental knowledge in the general case only with allowance for their content aspects (as in the arguments examined above bearing on definition of the process of analytical generalisation). If scientific laws possessing the characteristics of universality and necessity cannot be distinguished from empirical generalisations of a chance character (such as 'all my friend's dogs have a white patch on the brow') by purely

logical means in a general form, then, by explicating their essential differences at the content level, some of the explanations can then be described formally. *In the general case formal logic is always a necessary, but insufficient condition for analysis of such a kind.* That proposition relates as well to the apparatuses of modern formal logic now being successfully developed, which more adequately reflect the real practice of reasoning ('relevant logic'). This logic will seemingly be more adequate for purposes of logical analysis, but it will not transform formal logic into a universal means of dealing with methodological problems of cognition (knowledge) (on this see Chap. III, § 5).

The transition from concept $x(A(x) \wedge B(x))$ to concept $xA(x)$ follows the law of inverse relation between the content and the class of objects of a concept: the smaller the content of concept $xA(x)$ obtained by generalising the initial concept $x(A(x) \wedge B(x))$, the greater its class of objects will be:

$$Wx(A(x) \wedge B(x)) < WxA(x).^9$$

It can be said in a general way that if concept $xC_2(x)$ is a generalisation of $xC_1(x)$, then the following takes place:

$$(1) C_1(x) \vdash C_2(x), \text{ i.e. } \forall x (C_1(x) \supset C_2(x))$$

is logically true;

$$(2) \text{ moreover, } \forall x (C_2(x) \supset C_1(x)), \text{ i.e.}$$

$$\exists x (C_2(x) \wedge C_1(x)) \text{ is true.}$$

Condition (2) means that the content of the first concept (that being generalised) is not part of the content of the second (generalised) concept. In other words, by virtue of the law of the inverse relation between the class of objects and the content of a concept, the class of objects of the second (generalised) concept is always broader than that of the one being generalised. In this way the concept of generalisation is linked with the concept of logical deduction.

Let us introduce forms of the generalisation of concepts¹⁰ that correspond to certain basic rules of deduction in the predicate calculus and are the full formalisation of certain partial cases of generalisation (on condition that we are able to distinguish them from the general case by their content).

(1) $x(A(x) \vee B(x))$ is a generalisation of the concept $xA(x)$ or $xB(x)$ (since $A(x) \vdash A(x) \vee B(x)$ and

$B(x) \vdash A(x) \vee B(x)$, by the rules of the introduction of disjunction. Example: 'A number that ends in 5 or 0' is a generalisation of the concept 'a number ending in 5'.

(2) $x A(x, a)$ is a generalisation of $x \bar{A} y A(x, y)$ (since $\bar{A} y A(x, y) \vdash A(x, a)$ by the rule of elimination of the universal quantifier. Example: 'the student who passed the examination in logic' is $x A(x, a)$ and 'the student who passed all his exams' is $x \bar{A} y A(x, y)$.

(3) $x \zeta y A(x, y)$ is the generalisation of $x A(x, a)$ (since $A(x, a) \vdash \zeta y A(x, y)$ according to the rule for introducing a quantifier of existence). Example: 'A man who knows a foreign language' is $x \zeta y A(x, y)$ and 'A man who knows German' is $A(x, a)$.

There can also be varieties of these forms.

The content and range of the initial concept are thus altered during analytical generalisation of concepts.

In the course of the development of knowledge the initial generalisations may complete a transition from knowledge of the less general to knowledge of the more general, but the content and *logical range* of the initial concept will not be altered thereby. But the so-called *empirical range* of the concept will be altered; it will, for example, increase.

By the logical range of a concept is meant the class of objects (class, set) defined by the content of the corresponding concept. In this case, when the class of objects is given in tabular form, without distinct formulation of the concept's content, identification of the objects studied suggests their determination by the appropriate attributes of the objects that are included in the content of the corresponding concept. By the empirical range of a concept is understood the quantity or cardinal number of elements of the class of objects (class, set). If the number of elements is increased with an unchanged content of the concept, we speak of a change in its empirical range (although its logical range remains unaltered).

When a concept's empirical range increases (with invariance of its content and logical range), this is often spoken of as *expansion of the concept*, with which we shall often have to deal in the course of developing a concept. An example of a constantly expanding concept is that of chemical element at the various stages of development of our knowledge

of it: Mendeleev's periodic system was and is being enriched by newly discovered elements in conditions in which the concept of chemical element remains stable. A similar expansion took place during the development of the concept of elementary particle in physics.

When a certain new species of micro-organism is discovered, we include it in the class of objects of micro-organisms: the concept is broadened, although the logical range, which consists of the species of micro-organisms, and the concept of micro-organisms itself remain unaltered. The transition from sample to population is connected with the inclusion in both of one and the same quantitative concept $xP(x)$; I shall therefore treat this process as an expansion of a concept.

In the initial stages of synthetic generalisation, for instance, certain initial concepts (generalisations) are formed. Some objects are thereby generalised by identifying them in a concept $xP(x)$ according to certain attributes. The class of objects of this concept will be $WxP(x)$. In the course of knowledge we can *expand* the concept $xP(x)$ formed, and include newly discovered objects x in its range that are identified by property P both among themselves and with objects that are elements of class $WxP(x)$. Concept $xP(x)$ remains unaltered during this. A certain process, opposite to expansion of a concept, can be juxtaposed with it, viz., *localisation* of the concept, which is associated with the exclusion of certain elements from its empirical range while preserving its content and logical range intact.

The grounds for that may be the discovery that certain members of the range of the concept being examined have been illegitimately identified with its other members.

Whales, for instance, used to be included in the class of fish; they were called fish possessing such-and-such properties. At a certain stage of knowledge it became clear that they were not fish, and they were excluded from the set 'fish'. The concept of fish and its logical range were not altered by that exclusion. Similarly, *mutatis mutandis*, concepts of the expansion and localisation of various propositions and other thought forms (hypotheses, theories, etc.) can be cited.

With any application of a general judgment (or general concept, general theory) to a concrete situation, we are dealing,

generally speaking, with localisation of the said thought forms. We relate the content of a concept correct for each element of a non-empty set to a certain single member of it (individual): the predicate of the judgment, true for each element of the non-empty set, we relate to the corresponding individual. By employing a theory of sufficient generality, we apply it in practice to a certain individual situation. Such an application is localisation only when the concepts and propositions do not undergo any changes thereby (see Chap. II, § 5).

We constantly encounter localisation of judgments in deductive activity when, for example, the predicate of a general statement about a class of objects is transferred to an individual belonging to that class (for example, 'all liquids are elastic, mercury is a liquid, therefore mercury is elastic').

The difference between the logical and the empirical range that I have introduced and concepts based on a change in the empirical range are most important for the methodology and philosophy of science, and for scientific knowledge in general.

It is a matter of importance for description of the fullness and level of scientific knowledge, for instance, whether we include cancer in the class of objects of the concept of virus diseases and thus expand it, or exclude it from virus diseases and so localise it. The empirical range of a concept is often important for characterising the level of knowledge (for instance, the level of knowledge about elementary particles was different when the empirical range of this concept included only the electron and proton, and when it took in a huge set of newly discovered particles), and the level of social practice in mastering one sphere of reality or another.

Transitions in the process of knowledge from empty concepts to non-empty ones and vice versa, through a change in the concepts' empirical classes of objects, are vitally important as well for logical analysis.

Every expansion of a concept can be interpreted as a generalisation of the corresponding statement. The concept $xP(x)$ (those x that have property P) will then be interpreted as a certain initial inductive generalisation: each element of the set generalised in concept $xP(x)$ will have property P . Since the class of objects generalised in this concept grows during its expansion the generality of the statement increases in a cer-

tain sense: 'In each element of set x generalised in concept $xP(x)$, there is an inherent property P .'

The process of the *deepening (concretisation)* of a concept, by which is understood a growth of its content that does not lead either to a diminution or an increase of its class of objects (either logical or empirical), must be distinguished from its limitation or restriction by which growth of the concept's content leads to a diminution of its class of objects.

We meet deepening (concretisation) of a concept, for example, in the following cases.

(a) When the content of a concept includes a new attribute extensionally equivalent to an attribute or attributes already known and regarded as distinctive of it. The distinctive features of man, for instance, had already begun to be singled out in antique philosophy. The content of the concept of man usually included such traits, distinguishing him from animals, as 'being a rational creature', 'being capable of articulate speech', 'having religious feelings', 'having a sense of humour', etc. Benjamin Franklin's inclusion of such a trait in the concept of man as 'being capable of making tools' (and Engels' use of it, following Franklin) led neither to an increase in nor a reduction of its class of objects; the range of the concept of man was not altered. But the concept of man as a social being was radically deepened (concretised).

(b) When new species attributes, conceivable as linked conjunctively with the distinguishing one, are added to the content of a concept. When it became clear that the watermelon belonged to the gourd family the class of objects of the concept 'watermelon' was not altered, although the concept was deepened.

The processes of deepening (concretising) a concept are connected with the process of investigation carried out during the development of knowledge, and therefore have a synthetic character.

§ 2. The Logico-Analytic Generalisation of Statements. The Concept of Symbolic Generalisation

Any general statement can be regarded as a synthetic generalisation of facts in thought. We also speak of such generalisation

in connection with the process of statements' historical development. Generalisation of a statement can also be pictured as a special logico-analytic operation. Let us consider this in detail.

Assume that we have an attributive affirmative statement of the type ' S is P ', in which S and P are certain concepts. Then, we shall be dealing with *generalisation of a statement* of the type ' S is P ' (1) when we generalise concept S by the method described in the preceding section, while concept P remains unchanged; (2) when we generalise concept P , while concept S remains as before; (3) when we generalise both concepts. Generalisation of a simple statement is thus reduced to generalisation of its descriptive components.

Let me elucidate this by examples. (1a): 'Sodium is electrically conductive'—'metals are electrically conductive'. The second proposition will be a generalisation of the first if we have in mind that 'all metals are electrically conductive', and that any metal is electrically conductive. The following are also generalisation of statements (1b): 'all men are mortal', 'all living creatures are mortal'; (2) 'all tigers are mammals', 'all tigers are vertebrates'. In them the predicate of the second statement is a generalisation of the first. (3) 'All artificial earth satellites are artificial bodies revolving around Earth', 'all satellites of Earth are bodies revolving around it'.

The process of generalising initial true statements leads not only to true statements but also to false ones. Let us look at examples of this kind:

(1) 'all metals are electrically conductive'—'all substances are electrically conductive': a false statement has been obtained from a true one; or (2) 'all men are mammals'—'all living creatures are chordates'.

A logical criterion can be formulated establishing the conditions under which a true statement is obtained from a true one and a false statement from a true one.

(a) A true statement is obtained through generalisation from a true general statement when the new concept S' obtained by generalisation from the initial concept S does not exceed the initial concept P in class of objects. If S' exceeds the class of objects of the initial concept P , we get a false statement.

An example for the first case is 'all mammals are chordates'

and 'all vertebrates are chordates'. The vertebrates here (S) do not exceed chordates (P) in class of objects, and the statement is true.

An example for the second case is 'mammals feed their young with milk' and 'vertebrates feed their young with milk'. Vertebrates (S) here exceed the class of 'animals that feed their young with milk', and the statement is false.

(b) We will get true statements from any true general statement in which P coincides in range with S or exceeds it, in any generalisation of P : 'all mammals are vertebrates', 'all mammals are chordates'.

As we have seen, there is the following dependence for concepts: if concept $x A(x)$ is a generalisation in relation to concept $x (A(x) \wedge B(x))$, then there can be the deduction $A(x) \wedge B(x) \vdash A(x)$.

The analogous rule for generalising statements is formulated as follows. If a simple true statement C' is obtained by generalising separate descriptive components of the simple true statement C , then the deduction $C' \vdash C$ holds. In these cases the statement being generalised is deducible from the generalised one.¹¹

What has been said relates as well to the generalisation of laws. Such a regularity as 'for the class of threads K_1 made from very different materials, of different thickness, etc., there is a limiting weight w that the thread will support and beyond which it will break', can be regarded as a generalisation of the narrower law—'for a certain class of threads K_2 , of the same thickness, made from different materials, there is a limiting weight of 2 kg that these threads will support, but will break if it is exceeded', so that $K_2 \subset K_1 \wedge K_2 \neq K_1$.

It needs to be remembered that the obtaining of true propositions from false ones by the above means is not generalisation since the rules of logic cannot ensure in that case that a false initial statement C will be obtained from a true generalised statement C' . For instance, one can obtain the true statement 'all vertebrates are chordates' (C') from the false statement 'all mammals are herbivorous' by generalising its descriptive components. But the false statement 'all mammals are herbivorous' (C) cannot be obtained by the rules of logic from the true statement 'all vertebrates are chordates'.

I have examined how genus-species concepts and statements are generalised. The generalisations, as I showed above, have a logico-analytic character.

Systems of statements (including axiomatic ones) are also subject to generalisation. If a system of statements $A_1, A_2 \dots A_n$ is given in which $n \geq 2$, and if it is assumed that they are connected by a sign of conjunction, then the system can be treated as a generalisation of each of the statements A_1 . The complex, meaningful statement $A_1 \wedge A_2$ is a generalisation of statement A_1 (and equally of A_2). This is manifested in there being the deducibility $A_1 \wedge A_2 \vdash A_1$ (and equally $A_1 \wedge A_2 \vdash A_2$). Here we are dealing with the same conformity with law as in the case of generalisation of simple statements; the ones being generalised prove to be deducible through their generalisation from the generalised statement. The law of the inverse relation of content and range (class of objects) does not apply in the generalisation of statements; $A_1 \wedge A_2$ is a generalised statement as regards A_1 in the sense that more corollaries are deduced from the meaningful statement $A_1 \wedge A_2$ than from A_1 , i.e. the number of corollaries relatively deducible from it is greater than the number deducible from A_1 .

When an abstract axiomatic system is regarded as a concept,¹² then concept A_1 will, on the contrary, be generalised in relation to the concept of the system $A_1 \wedge A_2$, since $A_1 \wedge A_2 \vdash A_1$ takes place, no longer, however, as regards deducible corollaries but as regards the aggregate of models satisfying the system; the one axiom A_1 is satisfied by a larger set of models than the system $A_1 \wedge A_2$, since each new axiom, irrespective of the others, adds new requirements to the existing system and reduces the number of models that satisfy it. The question arises how to represent the system of objects described by a certain system of axioms in the form of a concept.

Assume that we have a certain set M of an arbitrary nature and a certain relation R defined by it. Then all the information about the pair of indeterminate objects (M, R) will be expressed by the following axioms:

- (1) xRx
- (2) $xRy \supset yRx$
- (3) $xRy \wedge yRz \supset xRz$.

The system of objects axiomatically described above can then be represented in the form of the following concept:
 $(M, R, x, y, z, \dots) \text{ } \forall x \forall y \forall z ((x \in M \wedge y \in M \wedge z \in M \supset$
 $\supset (xRx \wedge (xRy \supset yRx) \wedge (xRy \wedge yRz \supset xRz)))$.

This formula corresponds to the formula of the general structure of the concept:

$$(x_1 \dots x_m) A (x_1 \dots x_m).$$

Statements that are concrete in content with elicited logical terms are sometimes generalised by introducing variables instead of the descriptive terms. We then get a certain, identical, abstract structure for a set of concrete statements with a different logical content. Such generalisations can be called *sign-symbolic*.

For instance, we can write the structure of the concrete statement 'all metals are electrically conductive' in the following symbolic form by using variables for the descriptive terms: 'all S are P ', or ' $\forall x (S(x) \supset P(x))$ '. It may be said that we have written this sentence, concrete in content, in a generalised symbolic form which is thereby the singled-out structure of any generally held statement. These structures are the structures of any general simple statements that have a different concrete content. Symbolic generalisations of that kind can be applied to concrete simple and complex statements of different form.

So we pass from the statement (1) 'Elbrus is higher than Mont Blanc', concrete in content, to its structural expression $(x, y) R$, in which x , y and R are variables (if we interpret statement (1) not as attributive but as a statement with a relation). This form can be written in common language as: 'Something is to something else as something' in which the first and second 'somethings' correspond to the subject (object) variables x and y and the third to the variable for relations.

We employ this form of generalisation during the formalising of arguments when formulating logical rules.

By analysing contexts that are concrete in content and then generalising them symbolically we can solve a number of logical and linguistic problems, for example, formulate

the contextual definition of a *j*-operator. Or rather we define a *j*-operator by means of a *contextual definition in use*. (They were introduced by Hans Reichenbach).

A definition of a certain term that is made not in isolation from its context but by defining the context in which it is most often used is called a contextual definition in use. The term interesting us, and the context in which it is used, are defined by means of it. It is complicated, for example, to define the term 'metabolism' constructively, but much simpler to define the context 'metabolism is normal'. In practice we usually have to decide whether or not the metabolism is normal. Or, instead of defining the term 'the canonical form of such-and-such an expression', we define the context 'to put this kind of expression into the canonical form', and so on.

A *j*-operator is employed in the theory of *definite descriptions*, by which is understood the description of an object by its characteristics. For that the description must satisfy the requirement that the described object exists (uncontradictoriness of the description) and that of the uniqueness of the described object (fullness or completeness of the description).

The notation $jxf(x)$ is usually employed for a definite description, and reads: 'That x which possesses the characteristic f '. The *j*-operator (the operator 'that ... which') converts the function $f(x)$ into a certain object (term), or rather into a certain word designating the object.

The problem arises of giving the *j*-operator a contextual definition in use. The context in which it is most generally used is that of a certain single-term (unary) predicate g for which the description $jxf(x)$ is the argument. In other words we have to define the context

$$g(jxf(x)).$$

We begin development of the definition from an analysis of a context of concrete content. Take the sentence 'M.I. Kutuzov was born in 1745'. Let us write it symbolically: $P(x_1y_1)$ (1), in which x_1 is 'M.I. Kutuzov', y_1 is 'in 1745', and P is 'was born'. Instead of the proper name x_1 , we employ a definite description: 'the commander-in-chief of the Russian troops at Borodino'; $x_1 = (jx) C(x, z_1)$ (2), in which C is the commander-in-chief of the Russian troops and z_1 is 'at Borodino'.

Substituting (2) in (1) we get

$$P((jx) C(x, z_1), y_1). \quad (3)$$

Expression (3) can be read as: 'That x who was commander-in-chief at Borodino, was born in 1745'.

In order to define the definiendum (Dfd), i.e. expression (3) with a j -operator, we must make it conform to the expression Dfn, but now without the j -operator.

The Dfn for expression (3) can be written as follows:

$$\exists x \{ C(x, z_1) \wedge P(x, y_1) \wedge \text{Å}u (C(u, z_1) \supset (u=x)) \}. \quad (4)$$

Expression (4) records: 'There is an object x such as was the commander-in-chief at Borodino; he was born in 1745 and is the sole one'. The singularity is recorded in the expression $\text{Å}u (C(u, z_1) \supset (u=x))$ (5), which says that if the commander-in-chief at Borodino is u , he coincides with our x .

Note that $C(x, z_1)$ and $P(x, y_1)$ are in fact single-term predicates, since the variable x is connected in them (see expression (4)). Let us denote these predicates, concrete in their content, by distinct one-term predicates: $f_1(x)$ (x is the 'commander-in-chief of the Russian troops at Borodino', and $g_1(x)$ is 'was born in 1745').

Let us denote expression $C(x, z_1) \wedge \text{Å}u (C(u, z_1) \supset (u=x))$ by expression $\exists! x f_1(x)$ ('there is a single x who was commander-in-chief of the Russian troops at Borodino').

The whole of expression (4) will now have the form:

$$\exists! x f_1(x) \wedge g_1(x). \quad (6)$$

Expression (6) is the definiendum for expression (3).

The equality of expression (3) with Dfn (4), (allowing for the abbreviations introduced) can be written as follows:

$$g_1(jx f_1(x)) \equiv \exists! x f_1(x) \wedge g_1(x).$$

But g_1 and f_1 are predicates of concrete content.

By applying *sign generalisation* we get the definition of the j -operator in general form:

$$g(jx f(x)) \equiv \exists! x f(x) \wedge g(x).$$

In the sign generalisation examined we in fact complete the passage from the arbitrary context, concrete in content (a), which has a definite structure A , to all the contexts of concrete

content (to all x), that have the same structure A .

The structures of statements distinguished as a result of analysis (sign/symbol generalisations) can function at a certain stage of the development of knowledge as primary ones in relation to statements of the ordinary language of concrete content; we can form the latter by employing certain logical procedures on generalised sign structures. In such cases, as Karl Marx noted in his *Mathematical Manuscripts*, an *Umschlag in der Methode* ('a sudden change in the method') occurs: that which is genetically secondary may come forward as primary at a certain stage of knowledge.

The sign/symbol generalisation considered above on the basis of analysis of a separate sentence with concrete content differs from the sign generalisation obtained through generalisation of a whole aggregate of facts recorded in corresponding statements of concrete content ('protocol sentences'). Generalisations of this kind are the laws of the natural sciences written in the form of various mathematical functions with different content domains of their definition; they generalise aggregates of separate dependences between the meanings of the magnitudes being analysed, which are usually written down initially in corresponding tables. Such, for example, is the sign expression $pv = \text{const}$ (Boyle-Mariotte law).

Let me dwell on the case when the corollary, considered as a partial case in relation to the premisses, can be transformed into a generalisation of them. Say we have obtained a certain result from given premisses (for example, from a system of equations that may consist of one equation in the partial case).

Then the result, which is not equivalent to the system of premisses, but is a weaker statement than them, can be treated as a partial case of the system, and the system as a generalisation of this case. We can, moreover, obtain the initial system as a corollary from the partial case by the rules of mathematics and by changing the interpretation of the corresponding sign expressions. An equation of the second order can, of course, be written as follows in general form: $ax^2 + bx + c = 0$ (1). It will readily be seen that with certain fixed a and b ($a=1$, $b=0$), and also with $c = -d$, $x = z$, we get a new equation $z^2 - d = 0$, or $z = \pm \sqrt{d}$. With all these

conditions the equation is a partial case of equation (1) and is its corollary.

Now take $z^2 - d = 0$ as the initial equation. By substituting their partial values for z and d ($z = 2ax + b$, $d = b^2 - 4a$) we get equation (1). With such an approach it can be regarded as a partial case of equation $z^2 - d = 0$.

The substitution of $2ax + b$ and $b^2 - 4ac$ respectively for the variables z and d is substitution precisely of certain partial values, though from another field than that from which we derived the values for equation (1); in it this field of variables was of a lower order than z and d , and was that of individual objects (numbers).

Later I shall go on to consider other types of analytic generalisation.

§ 3. On a Generalisation of Concepts and the Theories in Mathematics Corresponding to Them

Analytic generalisations are characteristic of mathematics in which the original domain of objects is expanded by introducing new objects into it so that the laws of the theory that exist for the objects of the initial domain are retained for the expanded one. This generalisation relates in the first place to the relevant concepts, and also to operations with numbers and to the theories about objects in which these concepts figure. Such a generalisation is a means of increasing the algorithm potentialities of concepts and the corresponding theories. An example of analytic generalisation is the successive expansion of natural numbers to whole, rational, and real numbers that characterises the development of the mathematical system.

This generalisation of initial objects (concepts of natural numbers) is linked with transference to new objects and retention for them of the simplest laws of the arithmetic of natural numbers (of the type of identity relations), namely: the commutative law of addition $A + B = B + A$; the associative law of addition $A + (B + C) = (A + B) + C$; the commutative law of multiplication $A \cdot B = B \cdot A$; the associative law of multiplication $A \cdot (B \cdot C) = (A \cdot B) \cdot C$; and the distributive law $A \cdot (B + C) = A \cdot B + A \cdot C$.

The principal direct operations in the ordinary arithmetic of natural numbers, i.e. addition and multiplication, are performed without any restrictions. As for the inverse operations, i.e. subtraction and division, they are not always performable. The difference between two numbers $b - a$, for instance, is introduced by definition and signifies a certain natural number c such that $a + c = b$, i.e. c is the root of the equation $a + x = b$. But such an equation is solvable within the arithmetic of natural numbers if $b > a$. The expression $b - a$ when $a > b$ does not make sense here. In order to remove this restriction and give equation $b - a$ sense when $a > b$, the concept of a natural number is generalised and numbers introduced corresponding to $-1, -2, -3, \dots$ and simultaneously with them the definition

$$(b - a) \equiv -(a - b)$$

for the case when $b < a$.

Subtraction proves performable as well for $b - a$ with the introduced definition, when $b < a$. It is performed by subtracting b from a and placing the minus sign in front of the expression obtained. The subtraction operation becomes performable for all positive and negative numbers. The operations introduced for negative numbers must not violate the laws proper to the arithmetic of natural numbers. If we want to retain the distributive law $a(b + c) = a \cdot b + a \cdot c$, for instance, we must adopt the rule $(-1) \cdot (-1) = 1$. If we adopted the rule $(-1) \cdot (-1) = -1$, then, for $a = -1, b = 1, c = -1$ we would get, by the distributive law, $-1 \cdot (1 - 1) = -1 - 1 = -2$, and according to the laws of multiplication by 0 we would get $-1 \cdot 0 = 0$. In order to obtain 0 in the result as well by the distributive law, we would need to exclude rule $(-1) \cdot (-1) = -1$, and adopt rule $(-1) \cdot (-1) = 1$. These rules about signs play the role of definitions that must be brought into agreement with the main laws of the arithmetic of natural numbers.

When extending the number domain, and generalising the concept of natural number, we must thus see to it that the initial simplest rules of the arithmetic of natural numbers are observed. This is an indispensable condition of correct, fruitful generalisation.

Fractions, which removed the restrictions that prevented performance of the operations of division, were introduced into arithmetic in a similar way. The expression $\frac{b}{a}$ makes sense within the context of the arithmetic of natural numbers only when a is the divisor of b . In other words the expression $\frac{b}{a}$ would be comprehensible when the equation $a \cdot x = b$ proves soluble, i.e. when there is a whole positive number that satisfies this equation. The introduction of symbol $\frac{b}{a}$ into mathematics as having sense and denoting a fraction represented assumption of the existence of a solution for any equation of the type $a \cdot x = b$, i.e. the validity of the equality $a \cdot \frac{b}{a} = b$. This equality has the character of a definition. The introduced fractions thus make the operation of division unreservedly performable, with the exception of those cases when there is a zero in the denominator, for example $\frac{3}{0}, \frac{10}{0}, \frac{0}{0}$, etc. In other words we get a so-called *normal definition*,¹³ in relation to which the requirement of existence and uniqueness is formulated.

This requirement will be observed in order to define the division $\frac{x}{y} = z \equiv y \cdot z = x$ (1), for instance, if $y \neq 0$. Then z will exist and, moreover, be unique. If that condition is not observed, a contradiction can arise. If x and y are replaced by 0, for instance, and z first of all by 1, and then by 2, we correspondingly get the valid equalities $0 \cdot 1 = 0$ and $0 \cdot 2 = 0$. By applying the *modus ponens* rule with these substitutions from right to left to the equivalence (1),¹⁴ we get the expressions $\frac{0}{0} = 1$ and $\frac{0}{0} = 2$. From that it can be deduced that $1 = 2$ (if the left-hand expressions of the identity are equal, then the right-hand ones are also equal). It thus proves that the arithmetical operations (addition, multiplication, subtraction, and division) are performable in essence without restriction of the generality and do not go beyond the limits of rational numbers.

The expression 'in essence' that I used in the preceding paragraph means that division is not definite everywhere. The absence of this definiteness is of a fundamental character. The point is that the operation of division cannot be extended to the whole range of rational numbers (i.e. made definite

everywhere), if we want to maintain the simplest laws for the operations enumerated above.

The types of generalisation examined can also be studied on a more general and content-rich plane, namely as generalisations by analysis of the sense of certain expressions arising during the development of science, in the course of the development of mathematical theories. Take, for example, the generalisation of the concept of multiplication of natural numbers by a fraction (an example considered in Euler's *Elements of Algebra*). The concept *multiplication of natural numbers* ($a \cdot b$) can be defined as follows: to multiply a by b means to repeat the multiplicand (a) as addend as many times as there are units in the multiplier (b).

Assume that we need to multiply some natural number, for example 3, by $\frac{3}{4}$ ($3 \cdot \frac{3}{4}$). The former concept of multiplication (as I defined it) cannot be employed for this purpose, because it makes no sense for the case of multiplication of a natural number by a fraction: it is impossible to repeat the number 3 as addend as many times as there are units in the multiplier, because the multiplier $\frac{3}{4}$ does not contain a whole unit. The question arises how can the expression $3 \cdot \frac{3}{4}$ be given sense. Euler started by retaining the commutative property of multiplication for the different numbers that are being multiplied by one another. In this case the expression $3 \cdot \frac{3}{4}$ can be written as $\frac{3}{4} \cdot 3$. The latter now makes sense in the light of the definition of multiplication I gave above; now the multiplicand $\frac{3}{4}$ can be repeated as addend by as many times as there are units in the multiplier, viz., the expression $\frac{3}{4} \cdot 3$ is equivalent to $\frac{3}{4} + \frac{3}{4} + \frac{3}{4}$.

In this case the concept of multiplication as it was defined for the multiplication of natural numbers has been generalised and can be employed for multiplying a natural number by a fraction. The content of the concept 'multiplication' has not suffered any essential change thereby (the definition of the concept of multiplication remains as before). The concept has been generalised by *disclosing certain new relations in the object domain*. By retaining the commutative property of mul-

multiplication we have started to regard expressions like $3 \cdot \frac{3}{4}$ and $\frac{3}{4} \cdot 3$ as equivalent.

Generalisation of the concept of subtraction can be treated in a similar way. Originally subtraction was defined in relation to natural numbers. The expression ' $a - b$ ' was sensible only on condition that $a \geq b$ (a was greater than or equal to b); by this it was meant that an equal could be subtracted from an equal and the absence of a number obtained (zero as a number on an equal footing with other numbers was introduced much later into mathematics). The expression ' $a - b$ ' was considered senseless if $a < b$ (a was less than b). How then can the expression $a - b$ be made sensible when $a < b$? This was achieved by introducing negative numbers and the relations between them spoken of above, into the object domain. The domain having been thus expanded, the possibility was obtained of generalising the concept of the operation of subtraction, which began to be employed with negative whole numbers as well as with positive whole ones. Different interpretations of negative (minus) numbers were given later.

Vygotsky, in the wake of certain mathematicians, justified the introduction of negative numbers into mathematics by the need of simplifying the computation processes that arose during the solution of equations. He pointed out that even Descartes called negative numbers 'false'.

Assume we are given an equation $7x - 5 = 10x - 11$, with one unknown. If we transfer the known quantities to the left of the equals sign and the unknowns to the right, we get (1): $11 - 5 = 10x - 7x$; $6 = 3x$; $x = 2$. But if we transfer the known quantities to the right and the unknowns to the left, we get (2): $7x - 10x = 5 - 11$. Equation (2) cannot be solved without the introduction of negative numbers. By introducing them we get:

$$-3x = -6.$$

In order to get the same result in case (2) as in case (1) we have to introduce a rule that by dividing a negative number by a negative one, we get a positive number, i.e. $x = 2$.

The generalisation of positive whole numbers to positive and negative whole ones was associated (according to Vygotsky) with the elimination of several difficulties arising during the

solution of equations and with simplification of the algorithm of their solution.

The rules of the operations with them (he writes) follow from the need to agree the results obtained by means of negative numbers with those that could have been obtained without them.¹⁵

Other concepts were generalised in the history of mathematics in a similar way.

The introduction of negative numbers into mathematics thus extended and strengthened its apparatus. In that connection it is pertinent to recall here Hilbert's so-called ϵ -theorem (epsilon-theorem). Its sense is formulated as follows: if we employ an abstract, hypothetical object (introduced by means of the ϵ -symbol, i.e. $\epsilon xP(x)$ 'that object x which has property P ') when proving a theorem, and if we can compare this proof with another in which the new, abstract, hypothetical object does not figure, then this object is lawful and can be employed. This theorem of Hilbert's justified the introduction of imaginary and complex numbers into mathematics. Its sense can also be elucidated in the example cited above of the use of negative numbers to solve an equation with one unknown and of solution of the same equation without the use of negative numbers.

By his theorem Hilbert validated the thesis that the apparatus of mathematics should be expanded in such a way that the object newly introduced by the ϵ -symbol could be eliminated in certain circumstances and the results obtained by means of it could also be obtained in a certain sense without it.

Other concepts, too, have been generalised in the same way as positive whole numbers were generalised into positive and negative whole numbers during the development of mathematics. We know from the history of mathematics, for instance, that the Greeks opposed unity to many, which meant that many could not consist of unity (of one object). The Greeks related number only to many. Hence it followed that unity was not a number in their understanding. Furthermore, zero was not regarded as a number in Greek (and later European) mathematics. Zero demonstrated the absence of a number. The course of the development of mathematics, however, indicated the expedience of regarding zero, as well as unity, as

a natural number.

Descartes' ideas about the solution of algebraic equations played a big role in establishing zero as a full number (which was associated with the transfer to it of the operations that had been performed with the ordinary numbers 1, 2, 3, 4, ...). He proposed writing algebraic equations in the form of an equality of a certain polynomial to zero. Such a writing was a necessary component of the general theory of the solution of equations that he created. It turned out, moreover, that the solution of an equation was equivalent to factorising the left side of the equation and equating each of the members of the factorisation to zero. This procedure was based on the simple truth that the product of two numbers could be zero if at least one of them were equal to zero. The job of making an algorithm for the solution of algebraic equations led to a need to regard zero as a full number.

Study and solution of the problems of set theory also led to a need to generalise the concepts of a subset and of a set. Any set M , of course, consists of elements a, b, c , etc. An element of it will be an object considered from the angle of its belonging to this set. The elements of set M will be objects possessing the same property as that by which it was formed. We can also form subsets of a given set from its elements. Thus, if set M consists of three elements— a, b , and c (i.e. $M = \{a, b, c\}$), then combinations of elements $\{a, b\}$, $\{b, c\}$, and $\{a, c\}$ will be subsets of M .

The concept of a set of subsets, and at the same time that of a set itself, were generalised during the development of mathematics. *The generalisation of the concept of a set of subsets* was expressed in the inclusion among the latter not only of subsets consisting of one element of a set, but also ones that embraced all the elements of the set.

The concept of a set was also generalised for cases when it did not include any of the elements; their number proved equal to zero (empty or null sets). The simplest example from which it follows that the raising of the algorithmisation potential of a generalised concept compared with the one being generalised is calculation of the number of subsets of the set by the formula $N = 2^n$, in which n is the number of elements of set M . If we want to know the number of non-empty subsets

of set M , we must employ the formula $2^n - 1$. If it is important to determine the number of subsets of set M from which one-element ones are excluded, we must employ the formula $2^n - 1 - n = 2^n - (n + 1)$. If it were necessary to obtain these answers without formula $N = 2^n$, i.e. in a more direct way, our reasoning would be more cumbersome.

The deduction of a means of calculating all the subsets of a set is based on a simple argument. Assume that we have a set M consisting of the following elements: $M = \{a_1, a_2 \dots a_n; b_1, b_2 \dots b_n; c_1, c_2 \dots c_n, \text{ etc}\}$. Any subset of this set may be characterised by what elements of the set it includes and what it does not. Since each element may be included in a given subset or not, there are two possibilities for forming a subset for element a :

- (a_1)— a belongs to the subset;
- (a_2)— a does not belong to the subset.

There are similar possibilities as well for element b of this set M . There are four altogether:

- (a_1b_1)—element a and element b form part of the subset;
- (a_2b_1)—element a does not form part of the subset, but element b does;
- (a_1b_2)—element a forms part of the subset but element b does not;
- (a_2b_2)—neither element a nor element b forms part of the subset.

Continuing this argument in the same vein, we must affirm that there are two possibilities for element c : it may form part of each of our subsets (a_1b_1), (a_2b_1), (a_1b_2), (a_2b_2) or may not. Consequently eight subsets are formed: ($a_1b_1c_1$), ($a_2b_1c_1$), ($a_1b_2c_1$), ($a_2b_2c_1$), ($a_1b_1c_2$), ($a_2b_1c_2$), ($a_1b_2c_2$), ($a_2b_2c_2$). We can conclude by way of mathematical induction that if there are n elements in our set M then the number of its subsets will be 2^n .

We thus find that among the subsets of set M_1 consisting of two elements, we will have subsets including both elements (a_1b_1) and also subsets that do not include either (a_2b_2). Similarly in set M_2 which includes three elements, there will also be subsets that include all three elements ($a_1b_1c_1$) and do not include any one of the three elements ($a_2b_2c_2$).

According to the formula deduced, $N = 2^n$, the number of

subsets of a set can be readily calculated when the number of its elements is unity ($n=1$). N is thereby equal to two (the cases (a_1) and (a_2)). The number of subsets of a set can be calculated from this same formula when the number of its elements is null; it has one set with a null number of elements, i.e. there is a single-element set, the sole element of which is an empty set. In that case $N=2^n=1$, which means that the number of subsets of the null set is unity. The introduced mathematical dependences of the $N=2$ type make the introduction of null sets into mathematics natural. Concepts of a null range ('empty concepts') began to be considered in logic before empty sets were in mathematics.

Subset and set (in contrast to the generalisation of natural number and other concepts examined earlier) were generalised more formally in science (allowing for the historical experience of the generalisation of concepts in mathematics) first with the aim of heightening the algorithmisation potential of certain formulas of set theory. The cases of generalisation of concepts described were simultaneously the means of generalising the corresponding theories.

§ 4. The Expansion, Generalisation, and Restriction of Axiomatic Theories

Let us first consider the simplest cases of the restriction (limitation) and generalisation of axiomatic geometrical theories from the history of mathematics, so as to pass later to the logically possible approaches to problems of the expansion, generalisation, and limitation of axiomatic systems.

At the beginning of the nineteenth century János Bolyai created a so-called absolute geometry, i.e. 'a geometry constructed independently of the postulate of parallel lines and therefore containing propositions common to Euclidean geometry and Lobachevsky's geometry'.¹⁶ Euclidean geometry is based on a postulate of parallel lines; in Lobachevsky's the postulate is replaced by its negation. On that basis Lobachevsky formulated his views on the relation of parallel lines.

The concept of parallelism itself exists in the geometries of Euclid and Lobachevsky but is absent in Bolyai's.¹⁷ The geometries of Euclid and Lobachevsky can be identified by that

attribute and at the same time distinguished by it from Bolyai's absolute geometry. The last is identical with Lobachevsky's geometry and that of Euclid in all its axioms except the postulate of parallel lines and its negation which figure respectively in the geometries of Euclid and Lobachevsky. The 'absolute geometry' therefore contains only those theorems that are common to the geometries of Euclid and Lobachevsky (see Fig. 1).

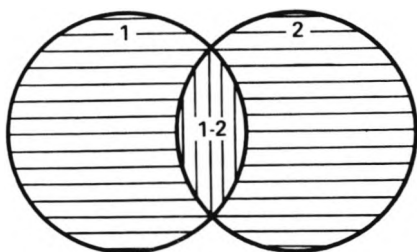


Fig. 1

Circle 1 in Fig. 1 signifies the set of theorems of Euclid's geometry, and circle 2 the set of theorems of Lobachevsky's; the horizontal hatching of circle 1 signifies the set of Euclid's theorems in the proof of which postulate V is employed (this set includes the postulate itself). The horizontally hatched part of circle 2 signifies the set of theorems of Lobachevsky's geometry that employs the negation of postulate V (and this set also includes the negation of the postulate itself); it follows from the figure that these sets are incompatible. The common part 1·2, hatched vertically, signifies the set of theorems that belongs at the same time to the set of theorems of both Euclid's and Lobachevsky's geometries and that arises when axioms and postulates common to the two geometries are employed. Part 1·2 is the set of theorems of Bolyai's geometry; it can be regarded as arising through limitation of sets 1 and 2.

If all these sets and subsets of elements (theorems) are regarded as the sums of elements joined by the plus sign (+) (or the U sign), then sets 1 and 2 are generalisations for set 1·2, which is their limitation. When 1, 2 and 1·2 are interpreted not as sets but as concepts (e.g. $xA(x)$, $xB(x)$, and $xA(x) \wedge B(x)$), i.e. as 'those propositions that have the

property of "being theorems of Euclidean geometry" ', 'those propositions that have the property of "being theorems of Lobachevsky's geometry" ', and 'those propositions that have the property of "being theorems of both Euclid's and Lobachevsky's geometries" ') then concept 1.2 will be simultaneously a limitation of concept 1 and concept 2.

When Lobachevsky was constructing his geometry, he did not simply exclude the axiom of parallel lines from Euclidean geometry, but replaced it by a new one that was a denial of Euclid's postulate V. Euclid assumed, for instance, that only one straight line parallel to a given straight line could be drawn through a point outside the latter. Lobachevsky not only denied that proposition but also stated that at least two straight lines that did not intersect the given line could be drawn through such a point (see Fig. 2).

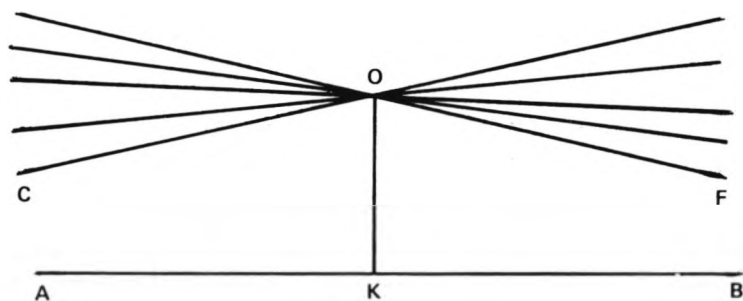


Fig 2

The straight lines OC and OF are parallel in relation to line AB (they do not intersect it when continued infinitely in each direction). Since two parallel lines can be drawn through point O that do not intersect line AB , an infinite number of straight lines can be drawn through that point which do not intersect AB (Lobachevsky, we must note, called only boundary lines that do not intersect a given straight line parallels, i.e. lines OC and OF).

In spite of the fact that Lobachevsky's axiom of parallel lines was the negation of Euclid's, Euclidean geometry proved to be the limiting case of Lobachevsky's, and the latter a generalisation of Euclidean. Lobachevsky's geometry passes into

Euclid's when the angles of parallelism (i.e. *COK* and *FOK*) are increased, tending to a right angle, and the lines *OC* and *OF* tend to merge into one. The original conditions are met in the limit; Lobachevsky's axiom of parallel lines passes into Euclid's axiom.

Now let us logically examine the possible way of generalising axiomatic theories from an example of generalising the propositional or sentential calculus to the functional or predicate one.¹⁸ The example given below is not an illustration of the historical path of the development of knowledge; there was never a state of affairs in the history of science such that the propositional calculus could have arisen first and been generalised later to the predicate one. Generally speaking, however, such a path of generalisation of axiomatic theories could also have been realised historically.

Assume that there is an axiomatic system consisting of $A_1 \dots A_n$ axioms *interpretable as concepts*. Then the disjunctive addition of a new, indeducible axiom *B* to the system leads to its generalisation (since there is the deducibility $A_1 \vee, A_2 \vee \dots \vee A_n \vdash A_1 \vee, A_2 \dots A_n \vee B$). Conversely, exclusion of axiom A_n from the system leads to its limitation. Then the initial system $A_1 \dots A_n$ will be generalised in respect of what we have obtained (since there will be the deducibility $A_1 \vee, A_2 \vee \dots \vee A_{n-1} \vdash A_1 \vee, A_2 \vee \dots \vee A_{n-1} \vee A_n$).

Voishvillo links this approach to an axiomatic system with the point that each axiomatic system can be regarded in essence as a definition of a demonstrable formula. For example, if a system has axioms $A_1 \dots A_n$ and rules $\psi_1 \dots \psi_n$, then the definition of a demonstrable formula will look as follows: (1) A_1 , or A_2 , or $\dots A_n$ is a demonstrable formula; (2) a formula deduced from other demonstrable formulas by any rule $\psi_1 \dots \psi_n$ (or combination of them) will be demonstrable.

With such an approach each formula demonstrable in constructive logic is also so in classical logic, but not conversely. The system of classical logic is a generalisation by comparison with constructive logic. The latter in turn is formed by the exclusion of certain axioms from the system of classical logic, axioms that are interpreted in this connection as concepts disjunctively linked together.

What I have said can be clarified by the following example.

Assume that we have a certain enumerative concept, similar to the concept of the demonstrable formula, $x (A(x) \vee, A_1(x) \vee \dots \vee A_n(x))$ (1). Then, (in accordance with the concept of the generalisation of concepts given above, in which the attributes are linked disjunctively), we get it that concept (2) $(A(x) \vee, A_1(x) \vee \dots \vee A_n(x) \vee, A_{n+1}(x))$ will be the generalisation of concept (1) on condition that the domain of the meanings of x is identical in both cases; the class of objects of concept (2) is broader than that of concept (1).

In the transition from propositional logic to predicate logic we generalise the former. That also happens in relation to formulas (properly constructed expressions). Every formula of propositional logic, for instance, is a formula of predicate logic, but not vice versa, generally speaking; formulas are met in predicate logic that are not only constituted by propositions regarded simply from the angle of truth or falsehood (in conditions of abstraction from their inner structure) but are also constituted by ones with expressions specific for predicate logic.

Every demonstrable formula of propositional logic, moreover, becomes one of predicate logic, but not vice versa. Formulas that are not demonstrable in propositional logic are to be found in predicate logic. In this case the axioms of predicate logic $B_1 \dots B_n$, interpreted as concepts, are added disjunctively to the axioms $A_1 \dots A_n$ of propositional logic, also interpreted as concepts. Here the following deducibility is ensured:

$$A_1 \vee \dots \vee A_n \vdash A_1 \vee \dots \vee A_n \vee B_1 \vee \dots \vee B_n$$

In the general case set M of the demonstrable formulas of propositional logic is a normal subset $M \cup N$ of those of predicate logic, in which N is the set of specific formulas demonstrable for predicate logic.

Furthermore, when the transition from the propositional calculus to the predicate calculus is considered on the plane of their interpretation as *judgments or statements*, it is convenient to represent the axioms as *conjunctively* joined. Then the axioms of the propositional calculus will be deduced from the axioms of the predicate calculus, i.e. the less general will be deduced from the more general. And that means, according to the definition of generalisation of statements given above, that

the system with the greater number of axioms (for example, those of the type of the predicate calculus) will be a generalisation for the system of axioms that constitute its normal subsets (for example, axioms of the propositional calculus). The number of corollaries obtainable from the generalised theory with this approach will be greater than the number obtainable from the theories that are being generalised. The content of the generalised theory is greater in this sense than the content of the theories being generalised. Its class of objects will also be greater when the corollaries obtainable from it are counted as elements of its range. That is natural, too, since we are dealing in that case with the generalisation of statements and not of concepts. The theory being generalised can thus be regarded as a partial case of the generalised one.

When the generalisation of axiomatic theories is regarded on the *semantic, interpretational* plane, rather than on that of allowing for the corollaries and demonstrable formulas obtainable, the transition from a concrete meaningful theory with its formal structure, and with a distinctly expressed sole interpretation, to the formalised theory corresponding to it, with many interpretations, will be a process of generalising it. The elements of the range of the theory will then be its separate interpretations.

If we base ourselves on the various ways of writing concepts (see Chap. II, § 1), we can represent various mathematical structures³ in a generalised, abstract form.¹⁹ The concept of the structure of any axiomatic formal system (with a finite number of axioms), for example, without interpretation and descriptive terms, but with a definite logic, can be represented in the form of certain very generalised concept. The forming of it presupposes employment of formalisation and symbol generalisation.

Hilbert's formalised (non-interpreted) geometry, for example, can be represented as a concept of structure in the following form: $[M_1, M_2, M_3, R_1^2, R_2^3, R_3^2, R_4^2, R_5^1] B [M_1, M_2, M_3, R_1^2, R_2^3, R_3^2, R_4^2, R_5^1]$, in which $B [M_1, M_2, M_3, R_1^2, R_2^3, R_3^2, R_4^2, R_5^1]$ is a conjunction of axioms in which variables $M_1, M_2, M_3, R_1^2, R_2^3, R_3^2, R_4^2, R_5^1$ stand for the initial terms of the theory linked by relations B . M_1 signifies the class of objects called points; M_2 and M_3 respectively signify the class of straight lines and

planes; R_1^2 , R_2^3 , R_3^2 , R_4^2 and R_5^1 are predicates, the first four of which represent relations. Hilbert called them 'to be situated', 'between', 'congruent', 'parallel', and the last—the property 'continuous'. The superscripts on R indicate the number of places of the R relations. With this kind of structure in mind Bourbaki considered mathematics a science of structures.

Deductive theory is very often defined, following Tarski, as a non-empty set of propositions closed as regards deducibility. In the light of this generalised definition any propositions can be regarded as scientific theories together with the corollaries following from them. This definition is unacceptable on the level of methodology. The point is that Tarski introduced it as a *synthetic* definition.

But such a definition of theory should be in fact qualified as *analytic*; the term 'theory' was already employed before it was introduced as a definition, and its task consisted in the meaning of the term not diverging from that established before the definition was introduced. Deductive theories in this sense can be called quasi-theories.

§ 5. Generalisation by Locke's Rule and Its Experimental Analogues

In applied predicate calculus a rule of generalisation called after Locke is widely employed (its first formulation is to be found in his works). It is written symbolically as follows:

$$\frac{A(a)}{\forall x A(x)} \quad (1)$$

(if a has certain property A , when a is a parameter, i.e. any, though stipulated element of the set M being studied, then it is a property of all the elements of set M from object domain D).

This transition from a stipulated object a with property A to all objects with property A is fully justified because all elements of set M from domain D are generalised according to this property. Locke's rule is applied in deductive logical and mathematical sciences and is the rule for generalisation in the analytical, deductive sense.²⁰

Indeed, the theorem of the sum of the internal angles of a triangle can be proved in an individual figure (for the given case), since we take only those properties $P_1 \dots P_n$ of a triangle

into consideration during the proof that are stipulated in the definition of it, or those properties $P_1 \dots P_n$ (or property, when they are treated as conjoined by a conjunction sign: $P_1 \wedge, P_2 \wedge \dots \wedge P_n$) by which the original generalisation for all triangles was made even before an axiomatic theory arose. We abstract from its other characteristics, i.e. length of the sides, size of the angles, area, etc., both during the forming of the original generalisation and during the forming of its definition in explicit form (for example, at the level of theory), and during the proof of the given theorem. By abstracting from the specific characteristics of separate triangles and singling out their common, distinctive properties $P_1 \dots P_n$ (equating all triangles in them), we generalise them and make them indiscernible in these general and distinguishing properties.

Once such a generalisation has been made, the theorem of the inherence of all properties A in a triangle (they are formulated in the theorem) can be proved in an individual diagram for any triangle whatsoever. In that we base ourselves on the definition of a triangle, on the axioms and postulates of the theory, and on previously proven theorems and the rules of logic. During the proof, for instance, we may only pass from $A(a)$ to $\forall x A(x)$ when object a , which represents the parameter, is identified with the objects x of a certain set M from object domain D according to certain properties $P_1 \dots P_n$ by which these objects are singled out and form part of the scope of the original generalisations either at the level of pre-theoretical or of theoretical constructs.

These properties $P_1 \dots P_n$ are not only distinctive for objects of the given set but are also essential. The point is that during the forming of the abstract objects of mathematics (such as various geometrical figures and numbers), we discard everything chance and secondary in their content and generalise them by their essential characteristics. Essential and inessential properties are therefore not differentiated in abstract mathematical objects. If both a property S_1 and a property S_2 are equally distinctive for a certain class of objects M , they are equivalent as regards the demonstrability of certain theorems. So, irrespective of whether we define a square as an equilateral rectangle or as a rectangular rhombus, one and the same theorems will be proved in essence

in the respective theories about them.

When considering the question of full mathematical induction, and of generalisation by introducing a universal quantifier and Locke's rule²¹ and pointing out the limitation of Aristotle's syllogistics and of the works of the scholastics related to it, Yanovskaya writes:

Mathematics above all had a need for rules of derivation such as would give it the possibility of proving theorems about *any* objects of any kind, even when these objects were as many as desired and the theorem could be proved specially *for each* of them, but the proof for object *a* differs from that for object *b*, if *b* differs from *a*. How is it to be tested, however, that a theorem really can be proved for *any* object of any kind, if the proof is different every time, and the objects are infinite in number? In fact it is impossible to carry out an infinite number of proofs!²²

The way out of this situation is provided by the rule of perfect mathematical induction (by complete enumeration). It makes it possible to reduce the proof for different *a* to one that is constructed identically for all *n* when they occur for any stipulated *n* (i.e. to prove the 'heritability' of a certain property *P* for *n* + 1, as soon as it is proven for any stipulated *n*). Suppose the first premiss of this principle to affirm that 1 possesses a certain property *P*. The second premiss states that if any arbitrary number *n* possesses property *P*, then the next number after it *n* + 1 will also possess property *P*. If both these propositions are true, it may be concluded that all natural numbers possess property *P*.

During proof, the second premiss is converted from a conventional to a categorical (absolute) one.

This last method of proving general propositions, sometimes called 'Locke's rule', is the rule of 'generalisation', or the 'introduction of a universal quantifier', usually employed in the predicate calculus, that enables one to derive the demonstrableness of $\forall xP(x)$ (for all $xP(x)$) from that of $P(a)$. (It follows from the point that the theorem is proven for an isosceles triangle *ABC* that it is true for all isosceles triangles.)²³

Although the concept of Locke's rule and mathematical induction are similar, it must be noted that they cannot be identified. Mathematical induction is based on a number of mathe-

mathematical propositions that Locke's rule does not have to satisfy.

Inductive logic both in the form of *enumerative* induction and in that of *eliminative* induction has played an immense role in science, being employed in it as an organon or instrument for making discoveries. The procedures of enumerative induction, which are certain forms of generalisation from study of separate cases, and equally the procedures of eliminative induction, which represent forms of discovering causal connections between phenomena, were more or less comprehended by representatives of experimental knowledge, who were carrying on research in order to obtain new truths. Induction (including analogy) is widely employed in mathematics.²⁴

In relation to mathematics Locke's rule is a means of *analytic* generalisation; when we employ it we do not go outside the boundaries of mathematics (of its abstract, idealised objects and their definitions, and certain axiom-propositions and theorems). The *analogues* of this rule are *synthetic* procedures; they are applied to experimental knowledge and analysis of experience is a *sine qua non* of the generalisations formed in it.

As for *experimental activity*, certain analogues of Locke's rule can be formulated that play the role in science of an important instrument for obtaining new truths. *These rules are employed for purposes of generalising and substantiating information obtained in a single experiment for all cases when this experiment is repeated in the same conditions.* The generalisation obtained thereby is based, generally speaking, on analysis of a *single, standard experiment*.

The legitimacy of generalisations obtained through empirical analogues of Locke's rule corresponds to the degree of their validity, which in general is peculiar to general empirical statements of a law-governed character (at the phenomenological or theoretical levels of their consideration). In any case they are hypotheses of a high degree of probability. Engels considered the truths of natural science on that plane.

An *experiment* is a method of studying objects and phenomena by which we intervene in their natural state and development, creating artificial conditions that make it possible to observe not only the properties arising in them as a result (including those that are latent or hidden in natural condi-

tions), but also objects that are not observable in normal conditions. This method is often linked with artificial division of the studied object and the natural conditions of its existence into parts, eliminating some of them, and their conditions, and recording of the effects in connection.

The main objects and tasks of an experiment can be formulated as follows: (1) discovery of a certain new object of investigation; (2) the bringing out of previously unknown properties of the studied object or phenomenon; (3) disclosure of forms of interaction and causal connections between phenomena.

In order to tackle these tasks we employ certain *analogues of Locke's rule*, which are united by there being established in them a pattern of generalisation from a single standard case to a certain general statement valid for an unlimited number of experiments of a certain kind. In other words we are dealing here with inductive generalisation *from a single standard case to a certain general affirmation* valid for an unlimited number of experiments of a certain kind, i.e. with inductive *generalisation from the single to the general* (by-passing the particular).

The empirical analogue of Locke's rule can be written as follows for case (1):

$$\frac{Es^*(a^*)}{\bar{A}x Es^*(x)} \quad (II)$$

This schematic formula can be read as follows: if a concrete object a^* is observed (directly or indirectly) in the experimental situation Es (which includes certain fixed conditions), the same picture will be observed in all other cases of setting up this experiment (a^* is made specified by the experimental situation and by its direct or indirect effect on the experimenter's sense organs).

For case (2) this analogue of the rule can be written as:

$$\frac{A(a^*), Es^*}{\bar{A}xA(x), Es} \quad (III)$$

If any concrete studied object a^* reveals characteristics A in a certain given experimental situation Es^* , then these characteristics will be discovered every time the experiment is repeated.

Similar schemes of analogues of Locke's rule can be suggested as well for the various forms of experiment of case (3). Many

of the results obtained in it are generalised by the methods of Bacon and Mill.

It is important, in connection with the experimental analogues of Locke's rule, to draw attention to the following points.

(1) One and the same experiments are repeated, as a rule, in order to test their purity, the correctness of the measurements made, the fullness of the characteristics brought out by them, and not to satisfy oneself that the generalisation obtained from the first experiment cannot be refuted by a counter case.

(2) The characteristics of the studied object of interest to us may be described at first by the way they were obtained, and later be given a description of their essence at the level of theory.

(3) Objects $a^*_1 \dots a^*_n$ are sometimes discovered in various experiments, and then identified as one. Such was the discovery of oxygen. Priestley isolated it by various means, from different substances, and then made the identification of what had been obtained (he called the oxygen obtained by the various methods air with a smaller portion of phlogiston than usual).²⁵

Because of the theoretical loading of experimental work an experiment is not organised and conducted blindly. The experimenter relies in advance on the relevance of what must be taken into account and what must be discarded. Furthermore, experiments are often made in order to discover what has been predicted by the corresponding hypothesis. that, however, does not exclude chance discoveries in science.

The elements of experimental activity mentioned above are inherent in modern science, with the immense scale of the experiments it carries out, and with their high technical equipping and consumption of energy, as well as in classical science.²⁶

An example of the solution of a task (I) in the course of an experiment was Roentgen's discovery in 1895 of the radiation he called X-rays. At first it was done by scheme (II); subsequently scheme (III) was used to study the properties of the radiation.

The simplest case of the application of scheme (II) is the generalisation of the results of experiments obtained during the analysis of certain uniform aggregates (iron, copper,

oxygen, water, etc.) The generalisation obtained in the course of it is considered legitimate because it relies on an assumption that has justified itself in experimental practice, namely: 'all parts of an aggregate are uniform, and will therefore manifest themselves uniformly in the conditions stipulated'. In this connection the individual part of the aggregate a^* selected for experimental investigation in fact plays the role of a parameter.

Now consider a more complicated case of generalisation of the data of an experiment by an experimental analogue of Locke's rule. I have in mind experimental investigation of a limited number of versions of some phenomenon in order to comprehend it as a whole and to extend the results to all possible future experiments.

Starting from the classical law of addition of velocities, for instance, it might be supposed that the velocity of light will be different in various frames of reference. But experience has shown that this is not so. The following four versions of the experiment were then carried out: (1) when the instrument enabling the velocity of light to be measured with adequate accuracy was at rest relative to a source of light located in a certain given frame of reference, (2) when the instrument was moving with a velocity v to meet the frame of reference; (3) when it was moving with a velocity v in the same direction as the frame of reference; (4) when the instrument was travelling with a velocity v perpendicular to the light wave being propagated in the given frame of reference.

In all the situations, as with case (1), the velocity of light did not alter and was equal to c . Here we are dealing with a certain complicated scheme (III) of the application of an experimental analogue of Locke's rule: in it the invariance of a certain property A (constancy of velocity) for a^* (light) is recorded in experimental situations (1) — (4) that embrace all cases of the relation of the light source and the frame of reference.

As will be seen, we generalise a certain limited class of experiments covering cases (1) — (4), with stipulated frames of reference and velocities v , and conclude that the velocity of light *in vacuo* is independent in all frames of reference of the magnitude and direction of their velocity and is always con-

stant and equal to 300,000 km/sec. This statement is considered true in spite of the fact that only a limited set of cases of the frames, velocities v , and directions was investigated from all the possible ones. In this example incomplete induction is treated in essence as complete: the four experiments mentioned for each of the cases (1) — (4) is considered sufficient in order to take the generalisation formulated above as true for any $\vec{v}$ of any frame of reference, and for all future experiments of this kind.

When validating the generalisation formulated above scientists rely on the following assumption: 'the velocity of light c is constant for all velocities v (and directions) of motion of the light source, if c is constant for some (or several) fixed velocity v (and direction) of motion of the light source'. The relationship only of the quantitative characteristics c and $\vec{v}$ is generalised by means of this assumption (and so transitions of the quantitative changes into qualitative ones, i.e. paradoxes of the 'heap' type, are excluded). We also base ourselves on the assumption that light is propagated in a vacuum, that space is uniform, that there are no grounds for distinguishing velocities v on the plane interesting us from all other velocities, and so on.

Experimental analogues of Locke's rule are employed particularly often in the technical sciences, i.e. in building, production technology, etc. Scientists very often check abstract technical models and projects in the laboratory on single full-scale models and on several single specimens. After such a test the originals are created in mass production that should retain the essential properties realised in the model. In such a case we pass, as it were, from a single prototype A (a^*) that satisfies our views embodied in the abstract design (project) to a generalisation $\forall x A(x)$, when we take a decision to pass to mass production of the corresponding technical devices. We know, moreover, that property A in single models is such that it will become universal in the event of a successful check.

The necessity for 'full-scale modelling' is due to the fact that 'pure' scientific laws are organically united in technical knowledge with certain considerations of a pragmatic quality, e.g. with the profitability of the devices, the possibility

of exploiting them in various conditions, their adaptability to the conditions, and so on. The 'pure' laws are 'clouded' as it were by secondary considerations. As Figurowskaya has written:

Technical knowledge is always related to a certain artificially created system constructed on the basis of a synthesis of a number of scientific laws. The sphere of their operation is concretely employed substances and energy of nature...

This uniting of 'pure' laws with material carriers must therefore be provided for in technical knowledge, in which the said laws prove to be 'clouded' by a whole number of secondary circumstances.²⁷

Technical knowledge, tested and embodied in machines, technical devices, etc., that are successfully used in practice, is evidence of the truth of scientific laws and of the adequacy of allowing for a number of secondary circumstances of a pragmatic character. All the specimens of a series of machines and technical devices produced are generally no more evidence of this than any specimen selected from the series. In that connection the following statement of Engels' seems very profound to me:

the steam-engine provided the most striking proof that one can impart heat and obtain mechanical motion. 100,000 steam-engines did not prove this more than *one*, but only more and more forced the physicists into the necessity of explaining it.²⁸

Consideration of the experimental analogues of Locke's rule makes it possible to conclude that generalisation in the experimental and technical sciences, and generalisation from part of a uniform aggregate to the whole aggregate *is not realised by means of incomplete induction*. They are certain *special* inductive procedures that make it possible to produce generalisations through experimental analysis of only a *single* item.

The concepts of *parameter* (in the logico-mathematical sense) and of *variable* are employed in Locke's rule. Menger's considerations about these concepts are of interest.²⁹ In order to differentiate unknown and variable quantities he suggested being guided by the following explanations. In the sentence 'Any x belonging to the class constituted by 1 and -1 satisfy equation $x^2 - 1 = 0$ ' (1), x has the character of a variable. In the sentence 'find all the numbers x such that $x^2 - 1 = 0$ ' (2), x plays the role of an unknown. In sentence

(1) there is given a class of numbers each of which will convert equation $x^2 - 1 = 0$ into a truth; this proposition, conceivable in its generality, contains a certain affirmation. In sentence (2) there is no such affirmation; it contains a certain imperative, viz., to find concrete numbers that will satisfy equation $x^2 - 1 = 0$.

Variables and unknown quantities are linked together in mathematics. We employ unknowns in the proof of mathematical theorems. Then, having determined even one value of the unknown x that figures in the theorem, we complete the transition to the variable x . In order, for example, to prove the theorem $\sin^2 x + \cos^2 x = 1$, we select a certain fixed value for the unknown quantity x and prove the theorem for that value of x . We then make the transition to the variable x .

Parameters (indeterminate constants) correspond to a special type of variable quantities. Their specific nature is that they are simultaneously fixed constants and variables, since their fixed values may be different. Menger introduced the concept of two types of parameter. When speaking of the class of all pairs of numbers $1 + 2t$ and $1 - 3t$ for any t , for instance, we employ the letter t as a parameter in the first sense; in each case replacement of t by a certain fixed number leads to a definite numerical pair.

When we employ variables in conjunction with unknowns, we are dealing with the other sense of parameters. In this case replacement of the variable (as a parameter) by some number leads to a certain equation. Such are the parameters a and b in the proposition 'for any numbers a and b find the x such that $x^2 + ax + b = 0$ '. Here, by substituting the corresponding numbers 0 and -1 for a and b , we arrive at the problem of finding an x (i.e. the value of an unknown) such as will satisfy equation $x^2 - 1 = 0$. Menger called parameters of this kind problem-parameters.

Unknown and variable quantities and parameters are concepts of different forms of the generalisation of objects considered in connection with problems of designation, modality, and the application of logico-mathematical rules.

The introduction of a variable x for a domain of objects, for instance, is linked with ability to distinguish and generalise those objects in the domain that will satisfy a certain

symbolic expression (for example, an equation, or a propositional function in general) and convert it into a truth. These skills are formulated in the shape of narrative propositions. When we register a certain value of x considered in a certain context, or in the structure of some proposition (which assumes the value of a truth), but bearing in mind that it is one of such values, we are dealing with parameters.

§ 6. Analytic and Synthetic Statements and the Process of Generalisation

The problem of analytic and synthetic knowledge was originally posed by Leibniz and Kant as one of analytic and synthetic statements. Analytic statements include the following two forms, viz., logically true and analytically true ones, which require neither experiment, nor comparison with reality to prove their truth.

Logically true (L-true) meaningful statements are such as are true because of their form and the meaning of the logical constants found in them. Such, for example, is the proposition 'this tetragon is or is not a parallelogram'. The truth depends only on the logical constants 'or', 'is' and 'not'. Descriptive terms come into L-true meaningful statements in an *unessential way*, i.e. any other terms from our field of argument can be put in the place of 'tetragon' or 'parallelogram', and we will then obtain true statements. Such propositions acquire the form of identically true statements through formalisation.

2. *Analytically true* (A-true) propositions are those whose truth is established from the meanings ascribed to their descriptive terms; the meaning of the logical constants is also taken into account. In order to substantiate the truth of such propositions it is necessary to allow for the meaning of their descriptive terms as well as of the logical constants; the relation between the descriptive terms can be written in general form as corresponding meaning postulates and their truth does not depend directly on the circumstances of the reality around them. It is sufficient to analyse the language to establish the truth of such propositions. Such, for example, is the proposition 'If John is a bachelor, he is an unmarried man'. We

write this in the form ' $X(a) \supset UnM(a)$ ' (1) in which X is the predicate 'is a bachelor' and UnM 'is an unmarried man', and a is an individual constant.

To substantiate the truth of proposition (1) we formulate the following postulate of meanings:

$$\forall x (X(x) \supset UnM(x)). \quad (2)$$

It establishes the compatibility of the descriptive predicates X and UnM and their corresponding concepts in general form (for all x). In the light of postulate (2) proposition (1) cannot be false for any individual constant from the field of our argument; when the antecedents are true the consequent cannot be false.

Generally speaking such postulates can be formulated as well for any scientific laws that are synthetic judgments, for example, for the law that 'all bodies fall to the ground with an acceleration of 9.81 m/sec. (3) (it being assumed that there is no resistance of the medium).

Insofar as this proposition is universal and necessary, a meaning postulate can be written in the form of the implication $\forall x (B(x) \supset FF(x))$ (4), in which $B(x)$ is the predicate 'is a body', and FF is the predicate 'free fall with an acceleration of 9.81 m/sec. In the light of this postulate proposition (3) (allowing for certain conditions, including disregarding the resistance of the medium) cannot be false.

The existence of the meaning postulates formulated above thus does not provide a possibility of drawing a line between *analytic* (A-statements) and *synthetic* statements. The criterion for differentiating them suggested by Carnap, therefore, which is based on meaning postulates (for analytic statements) and on the impossibility of formulating such (for synthetic statements) cannot be substantiated with adequate consistency.³⁰

The criterion of this differentiation can be represented in the following way: *if the meaning postulates of this type formulated on the basis of a corresponding explanation of the descriptive predicates (concepts) can be converted into logically true (L-true) propositions of the first type, we shall be dealing with analytic propositions (A-propositions).* Such clarified meaning postulates can be regarded as

a substitution of the descriptive terms for the respective variables in the laws of logic (i.e. in identically true statements). If the concept 'bachelor' (the predicate 'is a bachelor'), for instance, is treated as an abbreviation for the expression 'an unwidowed adult and unmarried man' ('an unwidowed man $(x) \wedge$, adult $(x) \wedge$, unmarried man (x) '), then postulate (2) acquires the form $\check{A}x$ (unwidowed man $(x) \wedge$, adult man $(x) \wedge UnM(x) \supset UnM(x)$) (5). This postulate can be treated as the result of substitutions in the identically true statement $\check{A}x (A(x) \wedge B(x) \wedge C(x) \supset C(x))$ (6), which means that the concept in the antecedent of postulate (2) contains information that is also in its consequent. In other words the information of the concept of an unmarried man is contained in the information about bachelorhood.

Such a transformation cannot be made with synthetic statements since the information contained in the concept of the consequent is not part, in all conditions, of the information contained in the concept of the antecedent. In fact, the content of the concept about free fall in postulate (4) does not form part of the content of the concept of a body in all conditions. It is a matter here of the actual properties of the objects considered, i.e. of the properties inherent in them in all circumstances. The actual properties must be distinguished from the potential (dispositional) properties as certain capabilities manifested in them only in certain circumstances.

The property of 'falling freely' is not part of the concept of a body since it is manifested only in certain circumstances; not all bodies are in a state of falling. Therefore the P of proposition (3) cannot be included in this proposition on the basis of the general definition S .

This circumstance was noted by Kant in his reasoning about the nature of analytic and synthetic statements. He pointed out that the statement 'all bodies are extended' ('*Alle Körper sind ausgedehnt*') (7) was an analytic one, and 'All bodies are heavy' ('*Alle Körper sind schwer*') synthetic, for example, since they did not possess this property during free fall, while bodies always have extension. When formulating and defining the concept of a body, we base ourselves on this property.³¹

The meaning postulate for proposition (7) looks as follows:

$$\forall x (\text{body}(x) \supset \text{extended}(x)) \wedge \quad (8).$$

This stipulates only that the concept of a body and extension are compatible. If, then, from analysis of the concept of a body and clarification of its definitional characteristics, the meaning postulate (8) becomes L-true, then our proposition (7) can be considered analytic. We can transform it into an L-true proposition, for example, as follows: $\forall (x) (\text{exists independent of us}, (x) \wedge \text{possesses spatial localisation } (x) \wedge \text{extended}(x) \supset \text{extended}(x))$.

This means that proposition (7) is analytic. For the proposition 'all bodies are heavy' an L-true meaning postulate cannot be formulated. It therefore is synthetic. What has been said above can be repeated *mutatis mutandis* for propositions in which the formulated meaning postulates stipulate the incompatibility of the corresponding concepts.

Carnap justly stressed that, in order to substantiate the analytic truth of classification sentences of the type 'all red-headed woodpeckers have red heads', we must allow for the purely semantic aspect of the importance of the inclusion of *P* in concept *S* (i.e. in the content of the concept of red-headed woodpeckers). Only on those conditions can we be persuaded of the analytic truth of this proposition.³² With that approach the following propositions will be analytically true: 'all foxes are mammals', 'all foxes are vertebrates', 'all foxes are animals'. They will be true on the basis of corresponding verified meaning postulates.

3. There is a class of analytically true judgments with relations whose truth is substantiated in the general case not only by verified postulates determining the relationship between the meanings of the descriptive terms (and equally the meanings of the logical terms), but also by the postulates describing the logical properties of the relations. Examples of this kind of proposition are: $\forall x \forall y (x \text{ father } y \supset \sim (y \text{ father } x))$, 'if John is the teacher of Peter's nephew, then Peter is the uncle of John's pupil'.³³

Some writers (Quine, Reichenbach, and others) do not class statements whose truth is based on meaning postulates

as analytic, since, in their view, the available criteria for such a classification is extremely indeterminate and even tautological.³⁴

Alongside concepts of the analytic and synthetic (or rather the analytic and synthetic truths of statements) in the narrow (logical) sense of the word, which I considered above, there are also those of the *analytic* and *synthetic in a broader sense* (we often base ourselves on them when analysing generalisations put forward in the form of concepts and theories of a certain kind). At this level the difference between the analytic and synthetic is not strict. The synthetic is always based on direct experience and is substantiated by reference to it; the analytic is based directly on definitions formulated as a result of analysis of the terms existing in a language, on the basis of their introduction through the initial concepts of a quite rigorously constructed theory. An example of the latter is logico-mathematical knowledge in which the process of its forming, its use in order to obtain new truths, and its functioning and development, are based on accepted definitions relating to abstract objects and constructed on the basis of aims determined by the tasks of the science itself. This shows up in the point that, in order to develop the fundamental theories of 'pure' mathematics, we do not have to resort to experimental study of some part of the material world. Obviously there are different intermediate levels of knowledge between synthetic and analytic knowledge in the broad sense of the word that include analytic and synthetic elements. The problem of generalisations (general propositions) that express laws of nature is widely debated in connection with the problems of the analytic and synthetic in the narrow sense in the literature in other countries. These general propositions are written in the form of corresponding implications. The problem has been analysed in particular by Hans Reichenbach.

The laws of nature (they include so-called nomological statements) are not analytically true, as Reichenbach justly remarks. Nevertheless they are universally true in a certain domain, i.e. do not depend (in any case to a significant extent) on the concrete conditions of time and place in relation to this domain.

Ordinary material implication, according to Reichenbach, is successfully employed in mathematics, in which we do without direct appeal to experience. In the natural sciences (e.g. in physics) we deal with another type of implication, whose truth is determined by its relation to experience rather than by analysis of the structural features of a formula.³⁵ Propositions are met in the natural sciences that have an empirical meaning formulated from a direct appeal to experience. In that sense the propositions that register laws of nature have a *synthetic* character.

The law 'All metals expand when heated' (9) is such a nomological proposition. It can be written in the form

$$\forall x [f(x) \supset g(x)],$$

where $\forall x$ is a universal quantifier ('for all $x \dots$ '), $f(x)$ 'x is a metal that is heated', and $g(x)$ is 'x—expands' ('for all x , if x is a metal that is heated, x expands').

According to Reichenbach, laws of nature must meet a number of conditions: (a) they must be true (establishment of their truth is a matter of induction and experiment); (b) they must be universal, i.e. not be chance things depending on the concrete conditions of place and time (general propositions of the type 'all Peter's dogs have a white patch on their brow' are excluded from their number); (c) written in the form of propositions with an implication, they must be 'sensible', i.e. they must not be just a consequence of the falseness of the antecedent or the truth of its consequent (in particular, Reichenbach introduced a concept of exhaustibility in order to validate this thesis).

In spite of the existence of several elements of an analytic character in propositions about laws, they nevertheless remain synthetic. Suppose, Reichenbach explains, we try to interpret them as analytic judgments based on meaning postulates similar to Kant's analytic *a priori* judgments. For that we define the heat of a metal in the natural law cited above (9) as 'the chaotic motion of molecules'. It can be concluded tautologically from that definition that the heated metals will expand if the expansion of a metal is interpreted as a change in its kinetic energy, i.e. an increase in the intensity of the chaotic motion of its molecules. But, Reichenbach explains,

we can never be convinced that the established experimental fact (the expansion of a metal when heated) has been adequately interpreted by us at the level of theory; the latter is always based on certain hypothetical assumptions that change as knowledge develops. This change depends directly or indirectly on experience and on the totality of our knowledge. In the light of the changed interpretation, the former analytic relation between the heating and expansion of metals may not always hold. Its analytic character may consequently prove illusory.

We must bear in mind, when discussing the question of the analytic and synthetic, that the knowledge that mankind disposes of is synthetic in character; it originates from experience and is tested by it. That applies not only to experience proper but also, generally speaking, to mathematical knowledge. Engels pointed out that the basic mathematical concepts of number and figure, operation and relation have an empirical origin. But other concepts built up on them (e.g. imaginary numbers) are even originally introduced into mathematics in a non-experimental way; they are historically the result of free creativity and imagination.³⁰ *The division of the components of knowledge into the analytic and synthetic belongs to a later stage of the development of science, and is mainly a matter, moreover, of the difference in the ways of introducing concepts and ways of establishing the truth of one judgment or another, and of the character and degree of justification of the knowledge obtained through the application of various cognitive procedures, and of the determination of the development of scientific theories by direct experience.* The development of rigorous criteria for differentiating the analytic and synthetic at the philosophical level is a very complicated matter.

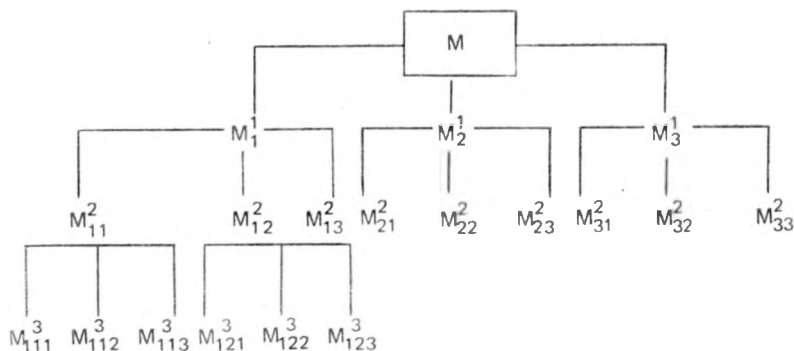
We encounter elements of the analytic in knowledge in the following cases:

(1) All knowledge contained in scientific theories and conceptions, and in mankind's experience, registered in language and systematised by language, is analytic in a certain sense, although it ultimately also has an empirical, synthetic origin. It is, as it were, a big handbook in which knowledge of the surrounding world is pooled. We draw diverse true information about the world from this 'handbook' apart from refer-

ring directly to the world. But this analyticity is relative since this 'big handbook' is being constantly amended by the development of science and perfecting of our experience. The 'handbook' thus includes both analytic and synthetic elements.

(2) Knowledge is analytic in certain rigid classification systems (for example, in the classification) of the animal and vegetable kingdoms), in certain systems of relations (of the kinship type) which are highly stable. The relations between the sets of studied objects are established dichotomously in classifications of that type. When considering the real differences between objectively existing sets we select those essential properties by which we distinguish them from one another.

Let there be a domain of M objects studied with the aim of classifying them. We distinguish a set M_1^1 (as a first level of division) and register its essential properties. Then from the not- M_1^1 we distinguish M_2^1 and record its essential properties, and so on until the set not- M_n^1 proves to be empty. Then we distinguish M_1^2 in M_1^1 according to essential properties and continue the subdivision as on the first level. The procedure is repeated until not- M_1^2 becomes empty. This process of classification is continued so long as non-empty regular sub-classes can be distinguished in the divisible set M^m . The result of this division can be represented in each concrete case as a tree.



The truth of each of the propositions like ' $M_1^1 \subset M$ ', ' $M_{11}^2 \subset M$ ', ' $M_{11}^2 \subset M_1^1$ ', ' $M_1^1 \cap M_2^1 = 0$ ', ' $M_2^1 \cap M_{11}^3 = 0$ ', is analytic; in order to substantiate their truth we usually resort to comparison of them with 'rigid', stable classifications.³⁷ These propositions express extensional relations between sets of objects. Analytic propositions corresponding to the given classification may also express intensional relations between the properties that determine such-and-such sets. We record the properties determining classes M , M_1^1 , M_{11}^2 , etc., respectively by PM , PM_1^1 , PM_{11}^2 , etc. Then the propositions ' $PM_{11}^2 \supset PM$ ', ' $PM_{11}^2 \supset PM_1^1$ ' are *analytically true in the intensional sense*.

All propositions that record properties corresponding to the subsets of each level of subdivision that are not included as sets recorded in our classification are *synthetic*. If, for instance, we subdivided all people by nationality, the existence of people born of mixed marriages will be synthetic as regards this classification. The statements 'Russians are not Germans', 'Frenchmen are not Spaniards' will, in turn, be analytic in relation to the accepted classification.

In order to characterise classifiable knowledge as analytic it is important to bear the following points in mind:

(a) The specific distinguishing properties essential for each member of the classification can be converted into a species distinction when defining a set corresponding to a member of the classification. A statement of equivalence, being analytic, corresponds to this definition.

(b) It is suggested that further experimental investigation of objects for which a rigid, accepted, classification has been established will not alter the already formed sets. If, for instance, we have called animals with a compound stomach ruminants in the system of classification, then we shall not be able to call newly discovered animals ruminants that have certain features of both a simple and a compound stomach. We can introduce a new classification term for them, or exclude them from the classification.

(c) When a system of units of measurement is being constructed, it is important to remember that, if some real thing

is presented during its affirmation and called a metre (defining the standard of measurement), such a definition will be analytically true (true by condition, by definition). A statement that a given rod is five metres long is already synthetically true, while the one that 'one metre equals 100 centimetres' is analytic.

When discussing the problem of the epistemological status of meaning postulates, Carnap considered them arbitrary and the result of agreement. He said, for instance, about the postulate of incompatibility of a bachelor and a married man ($\neg (Bx \supset \sim Mx)$), that its formulation was a matter not of knowledge but of decision.³⁸

Thus we see that it cannot be the task of the logician to prescribe to those who construct systems what postulates they ought to take. They are free to choose their postulates, guided not by their beliefs concerning facts of the world but by their intentions with respect to the meanings, i.e., the ways of use of the descriptive constants.³⁹

But even when it is a matter of very simple artificial semantic systems that include descriptive constants (not to mention bits of the ordinary language), and if, moreover, we hope to employ the systems purposefully so as to solve real cognitive tasks (and not simply as an exercise), the forming of the meaning postulates should be based on the meaning of the terms that have taken shape in the ordinary or scientific language. The forming of meaning postulates is therefore primarily a matter of knowledge and not of decision.

In this connection the division into analytic and synthetic rules and procedures employed in the process of cognition to obtain new knowledge has an important role. Procedures of this kind applied to premisses with a different content are called rules of inference, and are divided into deductive and inductive (probable) rules. The former are sometimes called rules of logic and the latter rules of generalisation. Rules facilitating the transition from true premisses with a definite structure (logical form) to true conclusions, also with a known structure, are called deductive rules of inference. Inductive rules (rules of generalisation) are those where the conclusion obtained when they are applied to true premisses, may prove either true or false. In order to substantiate or disprove the

truth of the conclusion a supplementary, experimental investigation needs to be carried out.

Some problems of inductive logic get their explication in probability logic, in which not only a true or false meaning is ascribed to statements but also intermediate meanings between 0 and 1. Such meanings are called the statements' degrees of probability, the probability of their truth, and the degrees of their confirmation. The apparatus constructed for probability logic makes it possible to make an approximate evaluation of the probability of statements whose truth is unknown.

All deductive rules are analytic procedures; knowledge not previously formulated in clear form is obtained through them by analysis of the premisses, without direct reference to experience and study of reality, as a result of applying the rules of logic to the premisses.

Inductive rules are synthetic procedures, since the question of the meaning of the results obtained by them is decided in the end by reference to an experimental study.

Deduction has usually been identified with synthesis in the history of logic, and induction with analysis, since we obtain conclusions through deduction in which knowledge contained in the premisses is synthesised, while during induction we select something common through analysis of the objects figuring in the premisses; this something common is represented as a result of analysis of knowledge of the individual. Engels stressed that analysis and synthesis are in fact inseparable in inferences.⁴⁰ By drawing a conclusion from the premisses of a syllogism, for instance, we both synthesise the knowledge contained in them in a certain sense and at the same time break some of the premisses down (and so analyse them). When speaking of analytic or synthetic procedures we evaluate them on another basis, viz., by their relation to experience.

Deductive rules can be subdivided into analytic rules proper, i.e. ones applied to premisses that are analytically true statements, and analytic rules in an improper sense, i.e. ones applied to arguments in which at least one premiss has been obtained on the basis of inductive rules (rules of generalisation) and whose truth has been substantiated by

a direct experimental investigation.

An example of the first rule is the following reasoning:

All vertebrates are animals

All mammals are vertebrates

All mammals are animals.

The specific feature of such arguments is that their premisses do not need direct verification. Their truth is ensured by the appropriate, accurately defined meaning postulates.

An example of the second rule is the following argument:

No cuckoo hatches the eggs it itself has laid

The bird in question is a cuckoo

The bird in question does not hatch the eggs it itself has laid.

Here the truth of the major premiss comes from applying the inductive rules of generalisation to the objects and from subsequent experimental investigation obtained by the inductive method of generalisation. The minor premiss is also a synthetic statement. By employing deductive rules in the improper sense in the process of cognition we obtain conclusions that are richer in information.

It is often said that mathematics is a realm of analytic knowledge. This proposition is substantiated by the fact that it is based not on real objects but on abstract, idealised ones that do not exist in material reality, but which are introduced by definitions at the level of mathematics. It is claimed that mathematics operates with constructs and not with reflections of reality. Such a point of view is held, for example, by neopositivists, who separate mathematics as knowledge based on agreements by a wall from science as knowledge based on direct sense experience.

Mathematics operates, in fact, with such concepts as a point, and defines it as something that has no measurement; it employs the concept of 'a straight line' and considers it an absolutely straight line having only length. There are no such objects in the material world. Mathematics also operates with numbers which (as I showed above in Chap. I, § 3) are abstractions of higher orders. But one must remember here that pre-theoretical and theoretical levels are distinguished in mathematics

just as in other sciences. At the first level the concepts of geometrical figures and numbers are not analytic but synthetic, since they have an origin in experience. Engels noted that circumstance, relying on the authoritative theoretical and historical studies of mathematicians.⁴¹

At the theoretical level the concepts of figures and numbers are introduced by definitions. The latter are treated in the context of theory as initial concepts unconnected with experimental investigation, as constructed by mathematicians on the basis of creation and imagination. These concepts already function as purely analytic. A jump is made in cognition that Marx called a sudden change in method (See Chap. II, § 2, pp 95).

The analytic character of theoretical mathematical constructs is thus relative, and is due to the fact that any scientific knowledge arises in man's material activity and is connected in some way or another with direct experience. Nevertheless this analytic character is a real fact, since the gap between mathematical theories and their original empirical basis is very great; it is much more considerable than the gap between the theoretical constructs of the natural sciences and their real basis.

¹ D. P. Gorsky. *Logica* (Logic), Uchpedgiz, Moscow, 1963, p 63.

² E.K. Voishvillo. *Ponyatie* (The Concept). Izd-vo MGU, Moscow, 1967, p 222.

³ See Voishvillo. *Op. cit.*, Chapter II, § 10.

⁴ *Ibid.*, p 117.

⁵ *Ibid.*, p 174.

⁶ *Ibid.*, p 222.

⁷ In science concepts like $x(A(x) \wedge B(x))$ are met in which $A(x)$ and $B(x)$ can be treated in a certain sense as specific distinctions; abstraction of one of them leads correspondingly to obtaining another generalised concept. Let x in our formula, for instance, cover the domain of plane geometrical figures, $A(x)$ mean 'being a rectangular figure' (having all right angles), $B(x)$ 'being an equilateral figure'. Such a figure is a square. By disregarding $A(x)$ we get a generalised concept $xB(x)$, 'equilateral geometrical figures', and by disregarding $B(x)$ we get the generalised concept $xA(x)$, 'geometrical figures in which all the angles are right angles'.

In the concepts $x(A(x) \wedge B(x))$ of the form described both $A(x)$ and $B(x)$ will be deducible from $A(x) \wedge B(x)$, and the concepts

$x A(x)$ and $x B(x)$ corresponding to them in content will be generalisations for the concept $x(A(x) \wedge B(x))$. If all genus-species concepts were ones like 'square', and the interpretation above of the properties constituting them were assumed, then the formulation examined above of the symbolic representation of the operation of generalisation of concepts could be considered a *full formalisation of the operation of generalisation*. But we shall have to distinguish concepts of this form (of the type 'square') from other types. This example signifies, however, that certain partial cases of the general concepts being analysed can be fully formalised if we are able to distinguish these partial forms by their content from other types.

⁸ I have satisfied myself on this from the example of applying the theory of definition by abstraction to experimental knowledge. At the same time I am also convinced that this is linked with several difficulties that I have removed by accepting a number of strong idealisations.

⁹ The sign $\subset$ means that the class of objects of $Wx(A(x) \wedge B(x))$ constitutes a normal part of the class of objects of $WxA(x)$.

¹⁰ See E.K. Voishvillo. *Op. cit.*, p 223.

¹¹ It is taken here that each statement is not a generalisation of itself in spite of the fact that $C \vdash C$.

¹² See E.K. Voishvillo. *Op. cit.*, pp 265-269.

¹³ See D.P. Gorsky. *Definition*, Progress Publishers, Moscow, 1981, p. 113.

¹⁴ The sign of equality by definition ($\equiv$) can be replaced in this case by the sign of equivalence $\sim$ (see Gorsky. *Op. cit.*, p 109).

¹⁵ M.Y. Vygodsky. *Spravochnik po elementarnoi matematike* (Handbook of Elementary Mathematics), Nauka, Moscow, 1979, p 114.

¹⁶ *Bolshaya sovetskaya entsiklopediya*, 2nd ed., Vol. 1 (Moscow, 1949), p 33.

¹⁷ Since Bolyai excluded the axiom of parallel lines, he was compelled to treat three partial cases as corollaries, namely: when the sum of the internal angles of a triangle is greater than $2d$; when it is equal to $2d$; and when it is less than $2d$. He then excluded the first case from his geometry because it introduced an axiom that contradicted this case, viz., that the external angle of a triangle is greater than any internal one, not adjacent to it.

¹⁸ See D.P. Gorsky. *Voprosy abstraktsii i obrazovanie ponyatii* (Problems of Abstraction and the Formation of Concepts), Izd-vo AN SSSR, Moscow, 1961, p 318.

¹⁹ See E.K. Voishvillo. Certain problems of the theory of meaning from the angle of logic and epistemology. In I.S. Narsky (Ed.) *Problema znaka i znacheniya* (Izd-vo MGU, Moscow, 1969), pp 146-150.

²⁰ See D.P. Gorsky. The anticipatory character of the reflection of reality at the level of human cognition. In D.P. Gorsky (Ed.) *Praktika i poznanie* (Nauka, Moscow, 1973), pp 71-72.

²¹ See S.A. Yanovskaya. *Metodologicheskie problemy nauki* (Methodological Problems of Science), Mysl, Moscow, 1972.

²² *Ibid.*, p 250.

²³ *Ibid.*, pp 252-253.

²⁴ See George Polya. *Mathematical Discovery*, in 2 vols. (John Wiley

& Sons, New York, 1962, 1965).

- ²⁵ See Thomas S. Kuhn. *The Structure of Scientific Revolutions* (U of Chicago Press, Chicago, 1970), Vol. II; No. 2, pp 52, 53, 54.
- ²⁶ See P.L. Kapitsa. *Eksperiment. Teoriya. Praktika* (Experiment, Theory, Practice), Nauka, Moscow, 1977.
- ²⁷ V.M. Figurovskaya. *Tekhnicheskoe znanie. Osobennosti vozniknoveniya i funkcionirovaniya* (Technical Knowledge. Features of its Origin and Functioning), Nauka (Siberian Branch), Novosibirsk, 1979, p 115.
- ²⁸ Frederick Engels. *Dialectics of Nature* (Progress Publishers, Moscow, 1972, p 229.
- ²⁹ Karl Menger. "Variables, Constants, Fluents". In: H. Feigl, G. Maxwell (Ed.) *Current Issue in the Philosophy of Science* (Holt, Rinehart and Winston, New York, 1961), pp 304-313.
- ³⁰ See Rudolf Carnap. *Meaning and Necessity* (U of Chicago Press, Chicago, 1956), pp 222-229.
- ³¹ See Immanuel Kant. *Critique of Pure Reason*. Translated by J.M.D. Meiklejohn (J.M. Dent & Sons, London, 1934), pp 30-32.
- ³² See Rudolf Carnap. *Philosophical Foundations of Physics* (Basic Books, New York, 1966), p 260.
- ³³ Analytically true definitions, which are a clear (explicit) formulation of the meaning of terms already existing in the language in which they were originally defined independently of the introduced definition, for example implicitly or contextually, are sometimes classed as analytically true knowledge. They are true when the meanings of the terms before and after definition are extensionally equal, and are intensionally different definitions of one and the same thing. We shall not consider them here (see L. Borkowski. Über analytische und synthetische Definitionen. *Studia Logica*, Vol. 4 (Poznan, 1956); D.P. Gorsky. *Definition*, pp 34-36.
- ³⁴ W.O. Quine, Two Dogmas of Empiricism. *Philosophical Review*, 1951, p. 60; republished in Willard O. Quine. *From a Logical Point of View*, 2nd ed. (Harvard U.P., Cambridge, Mass., 1964), pp. 20-47; Wolfgang Stegmüller. *Wahrheitsproblem und die Idee der Semantik* (Springer Verlag, Vienna, 1957).
- ³⁵ In his *Elements of Symbolic Logic*, Reichenbach tries to characterise nomological propositions that record synthetic judgments expressing laws on the basis of extensional logic by employing concepts of connective implication and exhaustibility.
- ³⁶ See Frederick Engels. *Anti-Dühring* (Progress Publishers, Moscow,) p 49.
- ³⁷ Operation $\sqsubset$ means 'inclusion of a class in another class'; operation $\cap$ means 'overlapping of classes'.
- ³⁸ Rudolf Carnap. *Meaning and Necessity* (U of Chicago Press, Chicago, 1956), p 224.
- ³⁹ *Ibid.*, p 225.
- ⁴⁰ See Frederick Engels. *Op. cit.*, (and 'Notes to Anti-Dühring').
- ⁴¹ *Ibid.*

Chapter III

PHILOSOPHICAL AND METHODOLOGICAL PROBLEMS OF GENERALISATION

§ 1. The Process of Generalisation and Various Forms of Expression of Concepts

In the course of synthetic generalisation we try to single out the general and distinguishing attributes (P) in the objects studied and to identify the latter by them. The process leads in this case to the forming of corresponding concepts. The P characteristics are intensions of the concepts formed. The set of objects of the domain possessing P characteristics constitutes the extension or range of the concept. With this interpretation of concepts we discard certain features of their extensional characteristics which may be different in various cognitive situations, and we examine the concept in a very general sense. By drawing attention to these extensional characteristics we get an opportunity not only to concretise them but also to bring out their relations with the P intensional properties.

The features of the extensional characteristics of a concept and the different relations of the latter with the generalised intensional characteristic P can be brought out in the forms of the concept $P(x)$, $xP(x)$, and $P(a)$, in which P is a predicate concrete in content. What is common in these forms of the concepts is that studied objects of the subject domain are identified in a certain way according to the property P common to them, which serves at the same time as their distinguishing property (and sometimes as their essential one).

The historical process of the formation of such concepts is complicated and dialectically contradictory. The concepts created by application of abstraction by identification are not absolutely stable; they are altered, clarified, and developed. But at certain, more or less long stages of historical

development concepts may be treated as relatively stable and invariant from the standpoint of their intensional and extensional characteristics. In such periods of the development of science concepts and what they apply to, and the relation of their extension and intension, are made more precise.

In this case we are treating a concept primarily in the role of a *propositional function*. The expression $P(x)$, we know, corresponds to a single-term propositional function 'x is P'. A function of that kind refers either a truth or a lie to any meaning of its argument. In that case they are interpreted as concepts.

The interpretation of propositional functions as concepts was first proposed by Frege. A set of the objects that satisfy a certain domain of objects D can be singled out from the latter by means of these functions (by substituting the corresponding names of the objects of the domain for the variable x), i.e. the objects in which a certain concrete property P is actually inherent and which form the class of objects or extension of a concept with an intension P .

Thus a set of numbers 2, 4, 6, etc. can be singled out from the domain of natural numbers by means of the propositional function 'x is an even number', since the statements '2 is an even number', '4 is an even number', '6 is an even number', etc., are true. Clarification of a class of objects by the method described sometimes presupposes solution of much more complicated problems; one very often has to clarify whether the newly discovered objects are related to the concept $P(x)$ or not. For that reason disputes arise among scientists that may go on for a long time. Similar problems arise as well in the course of extrapolating corresponding concepts.

Formula $P(x)$ thus expresses the structure of a concept that does not presuppose fullness of the selected class of objects when it is operated with. It corresponds to the form of a concept that plays the role of a rule enabling the class of objects or range of the concept $P(x)$ to be formed constructively, and its class of objects to be examined in the course of its constructive formation. Many concepts (for example, of metals, plants, and animals) were formed spon-

taneously during the historical development of knowledge. In some one of its stages we are dealing, as it were, with a 'secondary' moulding of the concept in conditions of a more constructive analysis of it connected with the solution of logico-methodological problems.

During the verifying analysis of statements (for example, scientific laws), and scientific theories, we often present corresponding concepts in the form of propositional functions.

When general concepts are being explained to students, for instance, by adducing many examples, they are essentially interpreted as propositional functions ' x is P ' and illustrated by means of objects that satisfy that function, i.e. the class of objects of the concept is formed again in the process of education so as to make its content clear to the students.

Concepts corresponding to the expression $xP(x)$ (i.e. 'those x that have property P ') are employed when the class of objects of the concept $WxP(x)$ is taken to be fully singled out in a certain absolute sense from the corresponding domain of objects. We usually interpret a concept that way when we define it analytically or introduce a new name for the class of objects given through description of their properties. We interpret the concepts in the sense of $xP(x)$, as well, when we establish the relation between them according to their class of objects, when we perform operations of limitation, generalisation, and division of concepts, and when we are engaged in classifying objects.

A concept of the type $P(a)$ is a concretisation of a concept with the structure $xP(x)$. It can be said of a certain class K (which can be written in the form $xP(x)$) that all its elements have property P , having in mind that whatever element we take will prove to have it, i.e. to sum up, each element will prove to possess this attribute.

The indicated generality of class K (i.e. that all its elements possess property P) can be expressed as follows: any stated individual element of class K can represent all its other elements ('the whole class K '). This means that a certain stated individual element of class K (let us call it a), which possesses property P , is substitute for any other member of it regarded from the angle of property P . In other words the sign a in the formula of concept

$P(a)$ functions as a certain *parameter*.

Concepts $xP(x)$ and $P(a)$ for instance are thus equivalent extensionally and intensionally; one and the same class K given through one and the same property P , corresponds to them.

When we want to illustrate the general through the individual we express it by means of concept $P(a)$. In the course of education we very often study general properties and laws from single objects and situations. School pupils, for instance, study the general features of rabbits, frogs, etc., from the example of concrete individuals.

Despite the fact that concepts $xP(x)$ and $P(a)$ are equivalent in the extensional and intensional senses, the forms of the connection between their content and range are different. On that plane they are not equivalent. This is expressed in the fact that use of concepts like $P(a)$, i.e. concepts in the sense of parameters, helps us single out certain connections in laws that would be inexpressible if we employed only concepts like $xP(x)$. A record in the form, for instance, of the formula of the law that the volume of a gas is inversely proportional to the pressure at constant temperature presupposes insertion of concepts into this formula in the sense of parameters. This law can be written as follows: $(P, v, p, t_a)(x)$, in which P is a variable ranging over different gases, v is a variable for different volumes of each of them, p is a variable for different pressures exerted on the gas, and t_a is the parameter for temperature.

§ 2. Extrapolation and Generalisation

By *extrapolation in mathematics* is meant a mode of approximate determination of the value of a function $f(x)$ at points x lying outside a segment $(x_0 - x_n)$, according to its values at points $x_0 < x_1 \dots < x_n$. On the quantitative level, systematic methods have been developed for determining the approximate values of points x , lying beyond segment $x_0 - x_n$, according to their values at points of it.

In *statistics by extrapolation* is usually meant extension of the results of processing sample data (obtained in relation to the studied part of a phenomenon) to an unobservable

part of the aggregate.¹ The indicators of the studied part are thereby regarded as approximations to the indices of the aggregate as a whole.

Results closer to the truth are obtained in the case when the interconnections are known between the extrapolated characteristics and the part of the studied phenomenon on the one hand, and interconnections between the part of the studied phenomenon and the unstudied part to which we are extrapolating something, on the other hand. We will get results closer to reality, for instance, from extrapolation of certain characteristics from the past to the future, when a link is established between these characteristics and a certain trend of development in the past about which there are no grounds for supposing that it will change in the future.

So-called *inductive extrapolations* are widely employed in scientific research, in which patterns established for one type of investigated objects, and for which a scientific explanation has been obtained, are transferred with certain corrections to other types (domains) of objects. Having explained, for instance, on the basis of induction, that certain gases possess a property *P* (e.g. compressibility), and having expressed this result as a quantitative law, we can then extrapolate this characteristic to gases not subjected to inductive investigation, bearing in mind that a special compressibility factor must be established for each of them.

In the *exact sciences* by *extrapolation* is often meant, as well, a method of mathematical hypothesis which consists in extending an equation describing a certain law of a studied field of phenomena to a new, as yet unstudied one, by assuming a change in the form, type and character of this equation.²

This method is widely employed in modern physics, and was also used in classical physics. With it the general form of the equation is often altered. That is what Heisenberg did, for instance, when he altered Dirac's linear quantum mechanics equation so as to obtain a fundamental law of the modern theory of elementary particles.

The general form of the equation sometimes remains unaltered but magnitudes of another nature are substituted in it. That is what Lorentz did when he tried to create an

electron theory employing Maxwell's electrodynamic equations. Schroedinger, while not in fact altering the general form of the classical wave equation,

altered the sense of the terms composing it, utilising the relation between the wavelength of the wave process and the impulse of the particle, unknown in classical physics, introduced by de Broglie. So Schroedinger's fundamental equation for the case of stationary states of the atom was obtained which underlies modern quantum mechanics.³

Several regulative principles are widely employed in extrapolations connected with a mathematical hypothesis, principles that are firmly based in physics and which must satisfy the modified equations, such as the correspondence principle, observation of a definite system of laws of conservation, and also

the requirement of invariance of the new equation in relation to the system of transformations considered obligatory for any physical theory in general, and the additional transformations specific for the given field of phenomena.⁴

Extrapolation is also employed at the qualitative stage of the study of an object; properties and characteristics inherent in one object field are extended to another, e.g. from the present and past to the future. Induction and sampling, analogy, and modelling are widely employed for that.

During the extrapolation of laws written in mathematical form from one field to another, the establishing of analogies between the fields plays an essential role. As a result of the discovery of analogies between the transmission of information and thermodynamics, for instance, to wit the laws of entropy, it proved possible to apply Boltzmann's formula, which establishes the link between the entropy of system S with the probability P of the system's state ($S = R \ln P$), to calculate the amount of information ($I = -R \ln P$).

Extrapolation is employed for prediction purposes, and to substantiate the extension of knowledge obtained, for example, within one science, to another science; it is widely employed to organise control of economic processes.

In the experimental sciences extrapolation is thus taken to mean the following:

(a) extension of qualitative characteristics from one object field to another, and from the past and present to the future;

(b) extension of the quantitative characteristics of one field of objects to another, and from one aggregate to another, using methods specially developed for the purpose;

(c) extension of certain equations to other object fields within the confines of one science, or even to other fields of knowledge; this is connected with a certain modification of them and with a reinterpretation of the sense of their terms (the method of mathematical induction).

Extrapolation differs from incomplete induction by simple enumeration. When the latter is employed, the use of special methods is not presumed, generally speaking, that heighten the probability of the conclusion from the given premisses; it is employed, furthermore, to investigate objects of one and the same field of objects. With qualitative extrapolation we usually transfer the characteristics interesting us in the course of the investigation from one object field to another (for example, in full-scale modelling the results of an analysis of not highly organised animals are extrapolated to man), employing certain additional rational considerations to substantiate this transfer, if no precise method had been developed.

When statistical methods are employed, certain quantitative characteristics are transferred from one aggregate (sample) to another, broader one (population). Statistical investigation of a sample is based on a specially developed method (mass observation, statistical grouping, generalised indices).

In cases (a) and (b) we are dealing with an extension of a certain concept $xP(x)$, in which P is the transferred characteristic. In case (c) concept $xP(x)$ transferred from one domain to another is usually altered; it is modified or some of its terms, at least, are interpreted differently. In the general case, therefore, extrapolation cannot be treated as an *expansion* of a concept. But if we extend equations to new domains, leaving them unaltered, we thereby expand the initial concept (for example, the concept of the set of phenomena described by the given equations). Extrapolation associated with a substantial change in the sense of the terms of the equation and extension of the phenomena described in equations often leads to the formation of new concepts about classes of objects that have the same structure (compare the extrapolation of the classical wave equa-

tion to cases of the steady state of the atom; the extrapolation of the laws of entropy to the quantitative characteristics of information).

The extrapolation described in points (a) and (b), and in the simplest cases of point (c) is sometimes connected with the generalising of corresponding statements (with generalising initial inductive inferences). With that the extrapolated characteristic P is interpreted as a predicate belonging to a certain original inductive generalisation. Thus, we may establish during an investigation that the experimental animals' consumption of a certain substance intensifies the course of a cancerous disease (P) in them. Then we begin to regard this property P (as a result of the extrapolation) as belonging (in this case in a hypothetical sense) to a broader set consisting of laboratory animals and a set of people.

Galileo's principle of relativity can also be interpreted in a similar way, though he formulated it only for mechanical phenomena. Einstein generalised it as follows: All laws in two systems of co-ordinates, moving uniformly and in a straight line relative to each other, are strictly identical, and there is no way of detecting the rectilinear and uniform motion. Here it is already a matter of all laws of physics.

The transition from a sample to a population can thus be regarded (a) as an extension of the initial generalisation, and (b) as a generalisation of the initial inductive generalisation. Case (a) was examined in § 7 of Chapter I. The initial generalisation of the results of analysis of the sample leads directly to an individual concept of the aggregate corresponding to the sample ($xP(x)$). Later this concept is extrapolated to another, broader aggregate, the population.

This process of extrapolation (case (b)) can be regarded, as well, as generalisation of an initial inductive generalisation, i.e. as generalisation of a certain judgment. The sample (aggregate) and population (aggregate) can be represented as certain sets (see Chap. I, § 7). Initially the predicate is stated about a set that corresponds to the sample ($xP(x)$). This predicate is then ascribed, by extrapolation, to a set corresponding to the population. Since the set of the sample constitutes a normal part of the set of the population, one

can speak of generalisation of the initial inductive generalisation (i.e. of the original judgment about the sample). The resulting statement is more general than the initial one; the latter can be deduced from it by the rules of logic.

§ 3. About the Alteration and Development of Initial Generalisations

The processes of the origin, forming, and establishing of initial generalisations (concepts) and inductive initial generalisations (statements) must be distinguished from those of their subsequent alteration and development. Here I shall consider these latter processes only in connection with generalisation and the dialectic of the analytic and synthetic.

Initial generalisations are altered and developed in various ways. Our knowledge about the objects being studied, reflected in these generalisations, can be developed, for instance, through the knowledge of the new and more profound properties which specificcate these objects, and by which they are generalised. There is usually a substitution of one group of specificating properties P_1 for another group of them P_2 relating to one and the same class of objects M . In this case P_1 is equivalent to P_2 : $P_1(x) \sim P_2(x)$. In other words P_1 and P_2 are extensionally equal. P_2 is often formed by substituting factual-explanatory descriptions for phenomenological ones at the level of theoretical constructs corresponding to the pictures of the world. The properties of gases to spread through all the accessible volume in the absence of significant external forces, and to acquire a corresponding density, for instance, are described on the phenomenological level and the level of the kinetic theory of gases. Properties P_1 and P_2 , moreover, often supplement rather than replace each other.

Sometimes, when altering and developing initial generalisations (concepts), we are dealing with their expansion (and not generalisation). That is how it is, for instance, when, during the development of knowledge, we include newer and newer elements in the class of objects of an initial generalisation about radioactive substances. The empirical class of its objects is thereby altered but the content of the initial generalisation

about radioactive substances does not undergo any substantial transformation.

In other cases the initial generalisations (concepts) are generalised. It has been established, for instance, during the development of knowledge, that light and radio waves have an electromagnetic nature and can be regarded as subsets of electromagnetic wave. Scientists deal with such a case of generalisation of the initial generalisation when, from study of coral polyps, they came to the conclusion that they could be divided into two sub-classes, eight-rayed and six-rayed. The concept of coral polyps thus began to be regarded as a generalisation in relation to eight-rayed and six-rayed coral polyps.

Thus, during the historical development of knowledge, *concepts are generalised* through the movement of thought both from sets to their regular subsets and from certain less broad sets to more extensive ones.

During the development of science new concepts may be introduced and old ones refined, enriched, or abandoned. The new ones are introduced as a result of deeper study of already known objects. There is often considerable enrichment of a concept through discovery of new properties in members of the former set, for example, the capacity of atoms to be transformed as complex systems into other atoms. As a rule concepts are abandoned in science that have been created, not on the basis of experiment, but in the course of speculative activity to explain the results of experience (the concepts of heat stuff or phlogiston, absolute space, absolute time, ether, etc.).

In the conditions of revolutionary transformations of scientific knowledge scientists come up against processes of the formation of new concepts, and clarification of old ones, and refutation of some of them, such as are of fundamental significance for science, are connected with transitions to new paradigms and 'pictures of the world', and with an essential change in and extension of the methodology of science (for instance, exclusion of the indivisibility and invariance from the content of the concept of the atom; the forming of a new generalisation about radioactive substances; elimination of concepts about ether from science;

introduction of the concept of a space-time continuum, and so on).

Initial inductive generalisations are recorded in the form of general statements. These generalisations are judgments that register regular connections between objects united in a certain class, and between their inherent properties, and may be altered in the course of historical development. For example, the statement 'all swans are white' was formulated through inductive generalisation of the facts. Later it turned out that it was not true; black swans were discovered in Australia. The first statement was thus refuted. That, however, did not mean that the generalising activity expressed in the formulation of the initial statement was useless. The discovery of black swans made it necessary to add a new species classed in the genus *Chenopsis* (*Chenopsis atrata*) to the six species of swans of the genus *Cygnus*, previously known. The discovery of black swans led, consequently, to a splitting of one concept into two in the logical sense; and in the biological sense two genera appeared, instead of one. Two generalisations were formulated instead of the first one: 'All swans of the genus *Cygnus* are white' and 'all swans of the genus *Chenopsis* are black'.

The refutation of general statements of a law character by contradictory examples thus means their concretisation (the introduction of differences into the identical, and the limiting of concepts) and not a cancelling out of the results of research. In the course of knowing connected with refutation of general propositions, our knowledge is not eliminated but is concretised; and certain corrections are made thereby in factual knowledge. That is how it usually is with the development of knowledge through refutation of general statements in which *S* (the subject) is not a fixed, completed concept.

During the accumulation of empirical knowledge we resort to definitions of new concepts and of the objects corresponding to them. In a certain sense such concepts become analytic; they are separated, in the course of operating with them, from the processes of their formation and begin to be regarded as existing only by virtue of the definitions. That relates primarily to concepts connected with acceptance of

such-and-such a picture of the world, to concepts about idealised objects, to concepts in the forming of which not only does direct generalisation of experience play a role but also certain speculative hypothetical considerations (such was Demokritos' concept of the atom).

Properties that prove specifying for the objects studied can be included in the content of the concepts corresponding to them. Sometimes, when classifications of the studied objects have already been developed sufficiently rigorously, the discovery of new objects not identical with those previously known leads to the formation of new classification groups of objects and corresponding concepts rather than to revision of the definitions of the previously formulated ones. It has been found in chemistry and physics, for example, that the most essential characteristics of elementary particles are mass and charge. These characteristics are also present in other particles, in different variations, and the particles have been classified according to the variation of these attributes.

It was originally supposed that the atomic nucleus consisted of protons only, i.e. particles with a mass of unity (P_1), and a positive charge, equal in absolute terms to the charge of an electron (P_2). The concept of a proton can be written symbolically as follows: $x(P_1 \wedge P_2)(x)$. This elementary particle has been defined by properties P_1 and P_2 .

Physicists subsequently discovered new elementary particles that form part of the atomic nucleus; one was discovered that was almost identical with the proton in property P_1 but differed from it in P_2 . It possessed a certain property P_3 , to wit had a zero charge (neither positive nor negative). P_3 proved incompatible with P_2 . This particle was called a neutron. Its discovery did not lead to revision of the concept of a proton (as happened with the atom when its fissionability was discovered) but led to the formation of a new class of elementary particles and a new concept of them: $x(P_1 \wedge P_3)(x)$.

Later the concepts of proton and neutron were generalised in that of a nucleon. In the theory of nuclear structure protons and neutrons began to be interpreted as different quantum states of one and the same nuclear particle,

the nucleon, which may be transformed into each other in the nucleus.

The difficulties accompanying the processes of empirical generalisation in connection with other forms of cognitive activity, and with extrapolation of the results of experience from the past and present to the future, were first investigated circumstantially in the history of philosophy by David Hume. He divided all inferences into demonstrative ones, studied by deductive logic (in which, according to him, relations between ideas were established) and moral or probable ones, relating to the generalisation of facts of experience, and to problems of the existence of material or ideal substances.

Hume considered it one of the most important tasks of philosophy to explain what underlay all conclusions from experience.⁵ He began his discussion of this by pointing out the difficulties of generalising empirical facts connected with problems of induction and causal explanation.

According to him the essence of inductive generalisation is to extend observations of objects of a definite kind with known properties and of their effects on other objects to all objects of that kind. The inductive conclusion (generalisation) obtained in that way differs essentially from the knowledge contained in experience and recorded in the premisses. The information contained in the conclusion exceeds that contained in the premisses (arguments).

In that connection Hume wrote:

These two propositions are far from being the same, *I have found that such an object has always been attended with such an effect*, and *I foresee*, that other objects, which are, in appearance, similar, will be attended with similar effects.⁶

And further:

In reality, all arguments from experience are founded on the similarity, which we discover among natural objects, and by which we are induced to expect effects similar to those which we have found to follow from such objects.⁷

'From causes which appear *similar*, we expect similar effects. This is the sum of all our experimental conclusions.'⁸

Experience yields information about the identity of properties, powers (forces), capabilities, and effects of certain

objects. 'When a new object, endowed with similar sensible qualities, is produced, we expect similar powers and forces, and look for a like effect.'⁹ But experience acquired in the past ceases to be the basis for conclusions about the future as soon as the natural order of things is altered.

If there be any suspicion that the course of nature may change, and that the past may be no rule for the future, all experience becomes useless, and can give rise to no inference or conclusion.¹⁰

Generally speaking Hume assumed that the world was not absolutely invariant. That being the case, statements could be conceived, without contradiction, that contradicted our experience obtained in the past. For, he wrote,

May I not clearly and distinctly conceive that a body, falling from the clouds, and which, in all other respects, resembles snow, has yet the taste of salt or feeling of fire? Is there any more intelligible proposition than to affirm, that all the trees will flourish in December and January, and decay in May and June? Now whatever is intelligible, and can be distinctly conceived, implies no contradiction, and can never be proved false by any demonstrative argument or abstract reasoning *a priori*.¹¹

(if, of course, propositions of the following kind were not taken as true assumptions in these arguments: 'all trees in our zone have flowered and will flower in May and June, and shed their leaves in December and January'). He therefore answered the question asked at the beginning ('What is the foundation of all conclusions from experience?')¹² by 'This principle is Custom or Habit'.¹³ He interpreted them, moreover, as extremely broad generalisations of experience converted into certain rules for the mind.

Hume was inclined to think that, for all our doubts, we must trust our experience and be guided by it in life. He saw the philosophical, logical difficulties of the process of understanding.

And though none but a fool or madman will ever pretend to dispute the authority of experience, or to reject that great guide of human life, it may surely be allowed a philosopher to have so much curiosity at least as to examine the principle of human nature, which gives this mighty

authority to experience, and makes us draw advantage from that similarity which nature has placed among different objects.¹⁴

But he considered knowledge quite unreliable as soon as it went beyond generalisation of direct experience, and it is in this that the scepticism of his philosophy is displayed. It is also expressed in his underestimation of causal explanations as a means of validating general assertions of an empirical character.

Dialectical materialism attaches great importance both to causal and other forms of explanation aimed at substantiating general statements of an empirical character (inductive generalisations), and to social practice, which vouches for the great role of experience. At the same time the philosophy of dialectical materialism does not recognise the invariance of the world, and does not make an absolute of the stability of its regularities and interactions. But there is no hint of scepticism in that.

In the examples adduced above I have shown that general statements of an empirical character can be refuted. These refutations (when they happen in the context of well developed science) do not render the refuted general statements (generalisations) useless. In such cases the refutations lead to a concretisation of knowledge, and foster its progressive development, which is linked with perfecting of the appropriate classifications and definitions. The limited character of empirical knowledge and the refutation of certain generalisations on the basis of examples that contradict previously obtained general statements, are not evidence of its deficiency but are evidence rather of a regenerative force and of its limitless possibilities for development.

During the development of mathematics there have been generalisations of certain dependences discovered through observation. In contrast to experimental physical knowledge the objects of observation in these cases are not real, material objects and their properties and relationships but abstract objects.

Polya cites the following example. Assume that we come across the proposition $1 + 8 + 27 + 64 = 100$. This could be written as follows: $1^3 + 2^3 + 3^3 + 4^3 = 10^2$. 'Now, it is

natural to ask ourselves: Does it often happen that a sum of successive cubes as $1^3 + 2^3 + 3^3 + \dots + n^3$ is a square?¹⁵

The basis for an affirmative answer is the discovery of a similar dependence in other cases:

$$1^3 + 2^3 = 3^2;$$

$$1^3 + 2^3 + 3^3 = 6^2;$$

$$1^3 + 2^3 + 3^3 + 4^3 + 5^3 = 15^2, \text{ and so on.}$$

This conclusion is rigorously proved in mathematics. As Polya pointed out:

In asking (our question), we generalize. This generalization is a lucky one; it leads from one observation to a remarkable general law. Many results were found by lucky generalizations in mathematics, physics, and the natural sciences.¹⁶

He noted that generalisations of certain, less common, concrete problems help to solve them more simply. That is connected with the fact that meaningful generalisations are associated with discarding the secondary that obscures the essence of the matter, and with singling out the essential that is relevant for solution of the given problem.

In contrast to corresponding scientific statements that are provable on the level of explanation and of a certain essential interpretation, mathematical statements are subject to logical proof (the principle of complete mathematical induction plays a major role in it). Such proofs are possible because of the specific character of mathematical abstractions; consequently the objects introduced into mathematics by way of definitions do not depend on direct experience.

An important problem of materialist dialectics is study of the forming of the general in 'living' nature, social life, and thought. A result of such study should be a concrete, historical typology of the forms of the general and of their patterns in nature and society. Karl Marx's theory of the development of the forms of value is instructive in this regard. He pointed out that historically the forming of equality relations (on the example of the direct exchangeability of commodities for one another) was a lengthy, complicated process.¹⁷

At the level of the simple (single, chance) form of value, the value of commodity *A* is expressed in its exchange

relation to a different commodity *B* (for example, 20 yards of linen exchange for one coat, or '20 yards of linen = 1 coat'). During the development of exchange the elementary or accidental form of value passes into a fuller, expanded form. It turns out, for example, that 20 yards of linen are not only equal to 1 coat but also to 10 lbs. of tea or 40 lbs. of coffee, or a quarter of corn, and so on. A certain range of goods is thus established that is the form of manifestation of the value of 20 yards of linen. The same can be said not only about 20 yards of linen, but also about any other commodity taken in a certain quantity and involved in exchange on the market. This form of value can be called particular.

It could have been shown that neither the simple nor the expanded form of value made it possible to draw a conclusion about the generality of relation *R*, i.e. that all commodities exchanged for one another on the market. That could only be done at the stage of social development when the expanded form of value was supplemented by the general form, i.e. when there were the following equalities: 1 coat = 20 yards of linen; 10 lbs. of tea = 20 yards of linen, and so on.

The development of commodity relations reaches its highest form of maturity when labour power becomes a commodity, i.e. under capitalism. It was under capitalism that the conditions were first created for elucidating the essence of value as that something common in all commodities that are exchanged for one another. In this case it can be determined through labour, the quantity of which is measured by the socially necessary labour time. Many phenomena of social life are developed according to the pattern: from the 'individual through the particular to the general'.

§ 4. From Generalisations to Their Application in Practice

During the mental movement from the general to the particular, and then to the individual at the level of mathematical knowledge,¹⁸ by means of operations of limitation, we pass from some classes of abstract objects to regular sub-

sets of these classes, and then to a set consisting of only one element.

Theoretical-cognitive and methodological difficulties arise with mental operations with abstract and idealised objects when we begin to employ them in practice (for example, in engineering), which is connected with their reification (i.e. conversion of the abstract and idealised into material things). These difficulties are overcome by including practice in the process of understanding, as a result of dialectical identification of the precise and the imprecise, the ideal and the material, the model and the original.

In the concepts that figure in empirical knowledge (I have in mind prescientific, everyday, and scientific knowledge), the passage from the general (through the particular) to the individual in thought, and from it to the individual in reality, differs in a certain sense from the similar process in mathematical knowledge. In empirical knowledge we are not dealing with sets of abstract objects (such as various numbers and geometrical figures), but are concerned with real ones. The latter are conjoined in sets by certain properties identical for them, which is linked with the transformation of material objects and their properties and relations into elements of the sets. These elements are treated solely from the standpoint of some properties common to them (i.e. are taken in the context of abstraction by identification). In empirical knowledge we are thus dealing with objects considered only in a certain interval of generalisation, i.e. generalisation according to certain common properties.

When sets of real objects, taken in a certain interval of generalisation, are considered as abstractions, they will be *abstractions of the first order*, while in mathematics we are dealing with abstractions of *higher orders*.

The question arises of the status of properties that do not come within the generalisation interval during identification. How are they to be dealt with? In the history of philosophy and psychology (and also in books on traditional logic) it is often stated in relation to such properties that we discard them, and ignore them, that we concentrate attention on the common, distinctive attributes by which

we single out the objects interesting us. These attributes are sometimes characterised as essential.

I suggest that the subject, at any level of understanding (prescientific and scientific), patently registers in concepts a certain complex of general and distinctive properties of the studied object (for example, in the form of definitions when it is a matter of theoretical constructs), and implicitly has in mind, in some form, the whole complex of properties of this object known to him. Rejection of unessential common and non-common properties of the studied object does not mean that they are completely excluded from the processes of thought but only that they are relegated to the background. What is implicit in thought can then become patent and be brought to the fore. This is manifested, for example, during the restriction of concept, when non-common attributes belonging only to regular subsets of a limited class are constantly included in the content of the initial concept.

At the level of empirical knowledge concepts and their corresponding sets differ essentially in many parameters from the concepts of abstract and idealised objects of mathematics. In mathematical knowledge, for instance, there is the principle of the equal value of extensionally equivalent definitions, which consists in extensionally equal definitions being indiscernible in the sense that, when added to the axioms of a theory and its previously demonstrated propositions, they make it possible to prove the same theorems extensionally.

Assume that we have two definitions: (1) squares are equilateral rectangles; and (2) squares are equiangular rhombi. These definitions are extensionally equal since they define one and the same class of objects, called squares. If addition of definition (1) to axioms A of theory T_1 and the previously demonstrated propositions of that theory, make it possible to prove the aggregate of theorems T_2 , and addition of definition (2) to axioms A of theory T_1 and its previously proven propositions makes it possible to demonstrate the aggregate of theorems T_3 , then T_2 is extensionally equal to T_3 and each of the T_2 theorems can be transferred to a T_3 theorem and vice versa.

Because of the extensional equivalence of the concepts

of equilateral rectangles and equiangular rhombi, the terms 'equilateral rectangles' and 'equiangular rhombi' are interchangeable in various contexts.

One and the same corollaries can therefore be drawn in logico-mathematical sciences from two extensionally equal definitions, but this cannot be done in the natural sciences, since the principle of extensionality does not operate in them in the overwhelming majority of cases, because they do not treat the processes of obtaining corollaries (theorems) from premisses in isolation from the processes of explanation, predicative force, triviality and non-triviality of corollaries, of the essential and the non-essential, experimental verification, etc. There is therefore no interchangeability of extensionally equal definiens in any context in two extensionally equal definitions (and corollaries drawn from them) when they are joined to the corresponding theorems. In the two extensionally equal definitions 'Man is a tool-making animal' and 'Man is an animal with soft ear lobes', for example, it is impossible to consider the phrases 'tool-making animal' and 'animal with soft lobes' to be interchangeable, which means that, in spite of the extensional equality of the corresponding concepts, *the principle of equivalence does not operate* in relation to them.

There are essences of various levels and orders in the content of the concepts of empirical knowledge; there is no *hard and fast* boundary between the essential and the non-essential, the stable and the unstable, the abstract and the concrete. Results obtained through formal logical, mental operation with concepts of an empirical content are therefore often not confirmed when applied in practice. Practice and analysis of the concrete content correct this process by intruding into it.

Assume that the concept of fluidity is restricted so that we can pass to such a form of liquid as water. Then we can pass to such a state of it as 'water at -5°C '. According to the rule of limitation any generic characteristics can be affirmed about water and true statements obtained thereby. It can be said of water, for example, that it is a liquid (that is a truth), but not about water at -5°C (that is false, since it is a solid in those conditions).

The laws of the natural sciences are always formulated with allowance for a number of conditions in relation to which they are valid. The law 'water boils at 100°C ', for instance, is fulfilled (not absolutely, but only with a certain accuracy) only in relation to ordinary conditions. With an unusual pressure (for example, high up in mountains) this law undergoes essential changes.

These conditions are sometimes never met in reality, and the law is formulated for certain ideal conditions. Kepler's law (in its limited formulation) that 'every planet moves in an ellipse, of which the sun occupies one focus', for instance, is fulfilled, strictly speaking, only when the orbits described by two heavenly bodies are investigated (in this case the investigated orbit and the Sun) in the absence of perturbing effects from other bodies, i.e. when the so-called two-body problem is solved. But in reality there are always such perturbations, and they are absent only in ideal (imaginary) conditions.

When some exact mathematical formula expressing a certain law of science is applied (not to mention approximate ones), we inevitably get approximate calculations in practice which is understandable, since all real means of measuring the magnitudes give them numerical values with ineradicable errors, which is connected with the complexity of the objects studied, the absence of absolute precision of measurement because of the imperfection of the instruments employed, and with the limited possibilities of our sense organs. Operation with approximate numerical results of measurements also yields an approximate result. The real measured situation is sometimes not sufficiently definite. We deliberately 'roughen' it and introduce certain terms that make it more definite. In that case, however, we do not obtain absolutely accurate results for the reasons mentioned above (that is how the distance between towns and the length of rivers, etc., are determined). Consequently, though the formulations of laws and their corresponding mathematical descriptions (formulas) are sufficiently accurate, the results of applying them are inevitably associated with inaccuracy and a certain inadequacy.

Theories are sometimes constructed in science in relation

to idealised objects rather than real ones (see Chap. I, § 6). Such is the kinetic theory of gases. It contains the concept of an ideal gas, in which the particles are represented as infinitely small spheres, characterised only by kinetic energy. It is considered, moreover, that the particles interact with one another only when colliding and that their potential energy is null.

The main laws derived in the kinetic theory of gases (Boyle's law, Gay-Lussac's law, Clapeyron's equations of the gaseous state) are valid, strictly speaking, only for an ideal gas. They characterise, with a certain approximation, only a gas that is in a rarefied state, when the difference between the volume occupied by it and the volume of particles is very great, which makes it possible to ignore the volume of particles, and equally the forces of interaction between them at a distance (the particles' potential energy of interaction). In other cases the application of these general laws is a more complicated matter in practice. For it is unlikely that a general method of applying idealised laws in practice to separate concrete cases can be formulated. In some situations empirically established factors are introduced into such a law, and they

may be in such a complicated functional dependence on the different states of the studied object that this way is rendered unsuitable in practice or suitable only in a few partial cases.¹⁹

One and the same object may be idealised differently depending on the methods of study (for example, classical or statistical). As Subbotin writes:

The main condition suggested for this, of course, is that the choice of method itself is dictated by the interests of objective, effective investigation of the studied object, and ultimately of its nature. An example is the application of different physical statistics (namely the Maxwell-Boltzmann, Bose-Einstein, Fermi-Dirac) to study of a gaseous state of a different nature and the forming in that connection of three different concepts of an ideal gas.²⁰

The elimination of forms of generalisation connected with the introduction of idealised objects into scientific theories is obviously attended with considerable difficulties. What we have abstracted from must be taken into account,

'made good', when such theories are applied in practice. Would it not be better in these cases to limit oneself to compiling appropriate instructions, practical recommendations, and formulary rules? Do we not do too much damage for the sake of systematism, algorithmic sophistication, information capacity, and heurism of theoretical constructs?

The creation of generalised, abstract, idealised theories is necessary for further progress of understanding.

The significance of the concept of an ideal gas is not limited by the fact that the laws of an ideal gas prove to be typical limiting laws for a real gas and express the state of the latter approximately in a range of small pressures and densities at temperatures sufficiently above the critical. It is no less important that investigation of the actual deviations in the behaviour of real gases from the laws of an ideal gas can yield certain phenomenological data that make it possible to judge the size of the molecules and atoms of a real gas and the forces of their interaction, although the question of the nature of these forces, and likewise of the law they are governed by, of course, remains an open one with that.²¹

When empirical scientific theories are being created, we sometimes do not introduce idealised objects into them, but resort to *idealisations as mental processes consisting in the use of assumptions in theoretical constructs*, in which we either eliminate (and especially abstract) or modify certain conditions of the existence of the studied objects, which leads to discovery of special, idealised, essential properties of this object in 'pure form'.

Marx widely employed this mental process as an important methodological tool in *Capital*. He showed, for example, that

the formation of capital must be possible even though the price and value of a commodity be the same; for its formation cannot be attributed to any deviation of the one from the other. If prices actually differ from values, we must, first of all, reduce the former to the latter, in other words, treat the difference as accidental in order that the phenomena may be observed in their purity, and our observations not interfered with by disturbing circumstances that have nothing to do with the process in question.²²

Marx stressed that the employment here of a mental operation representing a certain form of idealisation in accord-

ance with my interpretation had a direct basis in reality.

The continual oscillations in prices, their rising and falling, compensate each other, and reduce themselves to an average price, which is their hidden regulator. It forms the guiding star of the merchant or the manufacturer in every undertaking that requires time. He knows that when a long period of time is taken, commodities are sold neither over nor under, but at their average price. If therefore he thought about the matter at all, he would formulate the problem of the formation of capital as follows: How can we account for the origin of capital on the supposition that prices are regulated by the average price, *i.e.*, ultimately by the value of the commodities?²³

The average prices of commodities thus ultimately express the socially necessary outlay of labour on their production.

The idealisation employed by Marx, connected with ignoring the difference of the prices and values of commodities, and so leading to equating of them, is partly realised in the processes of the production and exchange of commodities.

Investigation of the laws of capitalist production, and of the social relations between people built up in the production of vital goods, is based on a set of such idealisations and idealising assumptions. Lenin remarked on this feature of *Capital*. In his article 'Once More on the Theory of Realisation'²⁴ he wrote that this theory developed by Marx in *Capital* was an abstract one explaining how the reproduction and circulation of social capital came about. Lenin criticised Struve for confusing the abstract theory of realisation with its implementing in any one country and at any one time.

Lenin pointed out several idealising assumptions ('premisses') on which Marx relied in *Capital*.

The essential premises of this abstract theory are, firstly, the exclusion of foreign trade, of the foreign markets. But, by excluding foreign trade, the theory of realisation does not, by any means, postulate that a capitalist society has ever existed or could ever exist without foreign trade. Secondly, the abstract theory of realisation assumes and must assume the proportional distribution of the product between the various branches of capitalist production. But, in assuming this, the theory of realisation does not, by any means, assert

that in a capitalist society products are always distributed or could be distributed proportionally.... The theory of value presupposes and must presuppose the equality of supply and demand, but it does not by any means assert that this equality is always observed or could be observed in capitalist society.... The theory of the average and equal rate of profit assumes, in essence, the same proportional distribution of production between its various branches. But surely Struve will not call it a theory of proportional distribution on these grounds.²⁵

In a letter to A. N. Potresov of 27 April 1899, Lenin also posed the problem of the relation of the abstract-theoretical and the concrete-theoretical in *Capital*. He defended the point of view that Marx's theory of realisation, and his solution in general of a number of conceptual problems, were based on certain idealising assumptions that were not realised in concrete cases.

'Concretely impossible' is not only realisation as put forward by Marx, but also land rent as put forward by him, and average profit, and the equality between wages and the value of labour-power, and much more besides. But the impossibility of something being realised in a *pure* form is not a refutation.²⁶

The laws revealed in political economy are realisable only on condition of the acceptance of a certain aggregate of idealising assumptions. The action of the law of value, for instance, is ensured by adoption of the following group of idealising assumptions: (a) the absence of monopoly in the economy; (b) the abrogation of any set-up that prevents transfer of labour power from one place of production to another, from one sphere of production to another; (c) labour in all spheres of production is reduced to simple labour, and the rate of surplus value is identical for all spheres of production; (d) the organic structure of the capital advanced is the same in every branch of production taken separately; (e) the length of the working day and money price are constant; agriculture is carried on like any other branch of capitalist production, so that absolute rent does not exist; (f) there is no merchant and bank capital; (g) exports and imports are balanced; there are only two classes; (h) the capitalist is constantly striving for maximum profit, and so on.

This all means that it is necessary, in practice, when applying a theory based on idealising assumptions, to eliminate those components of the theory that are introduced into it in the course of formulating the assumptions. In this connection Lenin wrote:

If Struve is bothered by the circumstance that 'perfect realisation is the ideal of capitalist production, but by no means its reality', we must remind him that all other laws of capitalism, revealed by Marx, also depict only the ideal of capitalism and not its reality.²⁷

In confirmation of that he cited Marx's own words: "'We need present,' ... 'only the inner organisation of the capitalist mode of production, in its ideal average ... as it were'".²⁸

The theories of complex objects, and above all social theories based on idealising assumptions, provide important advantages for understanding, for appropriate generalisations and theoretical constructs, and for singling out the most important and essential in the object of knowledge. They are constructed when the objects studied are very complicated. Theories constructed on such idealising assumptions rationally simplify the object studied, ridding it of the chance and secondary, and spotting its essence in distinct form. Understanding of the essence makes it possible to formulate the object's regularities and to reveal and substantiate the tendencies in its development in systematic form.

The application of idealised theories in practice in different cases of one and the same kind should be based on the dialectic of the particular and the general, of identity and difference. The organisation of the capitalist mode of production, represented in theory in an 'ideal average type', is by no means always realised with the assumed approximations, for instance; neither in all countries nor in any period of the history of capitalism. When this theory is applied in practice one has to remember not to apply the generalised theory to the individual situations as a template, but to use it concretely, allowing for its possible deviations from the 'ideal average type'. As Lenin wrote:

We do not regard Marx's theory as something completed and inviolable; on the contrary, we are convinced that it has only laid the foundation stone of the science which socialists *must* develop in all directions if they wish to keep

pace with life. We think that an *independent* elaboration of Marx's theory is especially essential for Russian socialists; for this theory provides only general *guiding* principles, which, *in particular*, are applied in England differently than in France, in France differently than in Germany, and in Germany differently than in Russia.²⁹

Such generalised theories, as I have already pointed out, make it possible to bring out and formulate what is most important and essential in the object studied, and in this case to substantiate the inevitability of the collapse of capitalism and triumph of socialism. With that in mind Lenin said:

Marx's conception of realisation inevitably leads to the recognition of the historical progressiveness of capitalism ... and, thereby, it not only does not obscure the historically transitory nature of capitalism, but, on the contrary, explains it.³⁰

When the kinetic theory of gases, based on the concept of an ideal gas, is being applied to separate cases of reality, we try, by introducing supplementary factors into the appropriate formulas, to make good what has been discarded. With that there is no need to develop or correct the theory on the grounds that it provides only an approximate kind of description of real situations. In the social sciences, as, generally speaking, in other sciences, where the object of study is developing and changing within the context of historical time, the relation between the theories and their application in practice becomes more complicated.

Before passing to a consideration of the complexity of this relationship, let us examine the concept of *astronomical time* and its varieties, *abstract-quantitative* and *historical time* more circumstantially. Every material process, we know, is characterised by duration. In order to measure the duration of processes differing qualitatively and quantitatively, so-called astronomical time is employed. It is measured by clocks, i.e. a certain periodic process that can be repeated many times, and moreover with the same duration. By comparing the duration interesting us with a clock (standard duration), and measuring it, we establish whether or not any two processes are equal in duration or whether one of them is shorter (or longer) than the other.

Abstract-quantitative time, as is known, does not presuppose preliminary recording of temporal and spatial characteristics of the studied situation (historical epochs and territories); these characteristics are also not included in the results of study of the situation interesting us. Assume that we are investigating the dependence between the volume of a gas and the pressure being exerted upon it. The results obtained will not depend on what historical epoch they were obtained in, or in what country. Physical and chemical laws operate practically unaltered in any historical age and in any area of the world. In any case the usual conditions of experiments, and the experiments themselves on the basis of which the laws are formulated, may be reproduced at a different time and at different points on the world's surface.

That time is considered historical for whose employment it is essential to fix the historical boundaries and spatial location of the situation studied. In the geological and geographical sciences, for instance, scientists have of necessity to include parameters of historical time when formulating certain statements and laws, indicating that such-and-such a position of things is taking place now (but may not have taken place at another time), and that another event occurred in such-and-such a geological era, period, or age, whose duration is usually measured in millions of years.

When biological and social laws of development are being formulated, a parameter of historical time is often included in them (or tacitly subsumed), i.e. the time of action of the laws concerned. In biology these parameters are also included in the action of laws of ontogenesis and phylogenesis. When processes of social development and civil history are being investigated, parameters of historical time come into the description of events as well as into the formulation of laws (explicitly or implicitly). Events are individualised by fixing spatial and temporal co-ordinates (co-ordinates of historical time), i.e. indications of what country, or what spot, some event or another took place in, and when. When social theories are being applied in which the situations studied are considered within the context of historical time, it is important to indicate their territorial loca-

tion (for example, to indicate whether the given laws operate in industrially highly developed countries or in less developed ones).

In practice, the process of employing general theories with idealised objects in which the objects are described in the context of abstract-quantitative time is obviously linked with considerable difficulties. The use of generalised theories with idealising assumptions, in which the objects are treated in the context of historical time, is associated with even greater difficulties. In idealised theories (e.g. the kinetic theory of gases) that study objects in the context of abstract-quantitative time, substantial corrections have to be made when they are being applied to separate real situations, but then the theory is maintained inviolable. In conceptions in which general regularities about objects are studied in conditions of historical time, new difficulties are added to the previous ones associated with the relation of the universal (in knowledge) and individual (in reality).

The fact is that the regularities relating to objects considered in the context of historical time have a strictly defined domain of application; they operate in concrete conditions of space and time. These laws are abstracted from situations, events, and phenomena that took place in such-and-such an interval in the past or are taking place in the present (but not in the future; to that field of being we simply extrapolate them). The specific character of the laws of social development, moreover, is that they are abstracted only from situations localised in certain conditions of space. The laws and specific character of the slave-owning socio-economic formation were studied by Marx and Engels, for instance, from the example of Greece and Rome, i.e. where they were realised in classical form. When social theories are being applied in practice to analyse the individual in reality, they are therefore constantly refined and developed.

Sometimes in the course of historical time the object studied alters so much that it is necessary to limit the operation of the former theory about it, and create a new one for the qualitatively new stage in the object's evolution. Lenin, for instance, created a doctrine about the

highest stage of capitalism, the theory of imperialism.

Imperialism (he wrote) emerged as the development and direct continuation of the fundamental characteristics of capitalism in general. But capitalism only became capitalist imperialism at a definite and very high stage of its development, when certain of its fundamental characteristics began to change into their opposites, when the features of the epoch of transition from capitalism to a higher social and economic system had taken shape and revealed themselves in all spheres.

Economically, the main thing in this process is the displacement of capitalist free competition by capitalist monopoly.³¹

When applying a social theory in practice we renew and develop it, as soon as the object studied itself alters and develops.

During the study of objects of the natural sciences, considered in the context of abstract-quantitative time, theories are developed through deeper knowledge of a stable object, which may be altered uniformly through transformation of the appropriate conditions. These changes are reversible; with restoration of the original conditions the original properties of the studied object are also restored. During study of objects of the experimental sciences, considered in the context of historical time, theories and conceptions are developed not only as a consequence of deeper knowledge of the object but also in connection with the need to allow for those of its characteristics that are manifested in it during the process of development and that are irreversible.

That is how things stand with application of certain concepts, statements, and theories, for example, in the social and biological sciences, and medicine. In them it is constantly necessary to clarify, alter, and develop ideas about known objects. In that regard the statement of the Soviet clinician Vasilenko is just: 'Medical work is inseparable from investigatory. Even the very process of making a diagnosis is a scientific quest.'³²

The difficulties arising when new generalised theories of social development are being applied to the particular in reality are deepened because the new stage of this development usually does not appear in pure form (especially when the time separating it from the previous stage is not

very great), but carries the imprint of the preceding stage. Apropos of this Lenin wrote:

Nowhere in the world has monopoly capitalism existed in a whole series of branches without free competition, nor will it exist. To write of such a system is to write of a system which is false and removed from reality. If Marx said of manufacture that it was a superstructure on mass scale production, imperialism and finance capitalism are a superstructure on the old capitalism. If its top is destroyed, the old capitalism is exposed. To maintain that there is such a thing as integral imperialism without the old capitalism is merely making the wish father to the thought.³³

Lenin gave brilliant examples of the application of Marxism in practice. He creatively developed the Marxian theory of socialist revolution in new conditions and drew a conclusion about the possibility of victory of the socialist revolution in one, relatively backward capitalist country taken by itself.

The features of the application of generalisations in practice in mathematical, natural science, and social knowledge mentioned above can be employed to develop the specific character of knowledge in different spheres.

Analysis of the process of passing from generalisations to individual situations in reality witnesses to the profound dialectic of knowledge. The idealisation and idealising assumptions that make scientific theories simple and demonstrable, and amenable to use of mathematical methods, makes them, at the same time, applicable in practice only on condition that we 'make good' what we have discarded, and that we eliminate the 'rigid' assumptions employed at the level of theory. This means that theoretical constructs and practice have a complex, dialectically contradictory relation to one another.

The difficulties of building theoretical constructs in the social sciences are also due to the complexity of substantiating the proposed abstractions in the subject studied, and the assumptions made. The point is that what we discard may subsequently prove essential to explain the essence of the matter, and that what we adopt when studying the object may, on the contrary, be inessential.

§ 5. Relevant and Irrelevant Generalisations
of Dependences.

Chance Generalisations and the Laws of Nature

We are constantly compelled, in the course of knowledge, to generalise dependences discovered in experiment and recorded in the form of tables and minuted statements. The information contained in them is true. The question arises whether it is possible to single out something general, regular, recurrent, and law-governed from the content of these tables and statements. Let us assume that the investigator answers that in the affirmative. Then how are we to write the established law-governed dependence in the form of a compact mathematical equation that will complete the generalisation of the dependence found?

Are there rules whose observance is a *sine qua non* for obtaining a relevant generalisation³⁴ of empirical dependences?

Let me define *relevant initial inductive generalisations*. By generalisations of this type I mean those that give a possibility, through parameters related to the essence of the matter, of reflecting the essential, regular, and law-governed. When generalisations are made from parameters not related to the essence of the matter (irrelevant ones), we will get irrelevant initial generalisations.

Consider an example.

Assume that we want to clarify the dependence of certain characteristics (y) of a certain substance on changes in its temperature (x). An appropriate experiment is mounted and it becomes clear that when the temperature (x) of the substance reaches 1, 2, 3, 4, etc., degrees, the characteristic (y) being studied receives the corresponding values of 2, 4, 6, 8, etc. The results of the inquiry are tabulated. Analysis of the experimental data recorded in the table leads us to the conclusion that the value of y will be double that of x for each case of the value of x . The result obtained is expressed by the following equation: $y=2x$, which reflects the dependence only between those parameters that have been investigated during the experiment made.

Suppose now that the experiment is repeated and yields

different results. When x has the values 1, 2, 3, 4 ..., etc., y has corresponding values of 4, 8, 12, 16, ..., etc. By generalising these data we get a dependence $y=4x$. This generalisation, too, does not contain any parameters unrelated to the substance of the matter.

Since the experiments have yielded different results it is natural to ask the reason for the difference. That is what the investigator does who is engaged in discovering and explaining the nature of regularities and laws of nature found.

Neopositivists (for example, Carnap) consider it illegitimate to pose the matter so and propose (from the position of a purely phenomenological and formal approach to the process of generalisation) taking further the generalising of the various regularities obtained by introducing parameters into the equations that have no relation to the essence of the matter.

A generalisation of the results of the two experiments can be obtained in a single dependence by introducing a new parameter z representing the number of the experiment (which has only two values: 1 for the first experiment, and 2 for the second). Then the general dependence covering the results in the first and second experiments can be written in the form of the equation $y=2zx$. Any value of y in both experiments can be calculated from this equation, and it will correspond to the value of x obtained experimentally.

Such a generalisation will not be relevant since it contains parameter z not related to the matter. It is external, extraneous, and inessential for the experimental generalisations originally obtained, i.e. $y=2x$ and $y=4x$. Irrelevant techniques of generalisation free workers from causal explanations of the phenomena studied, from disclosing their causal conditioning, and consequently, from penetrating to the essence of the phenomena studied.

If the recommendations of modern neopositivists were followed in science, generalisations created in the way described above would hardly be usable either in the course of development of sciences or in their applications in practice. Assume that someone who had undergone an operation

recovered, but next time died after a similar operation for the same illness (in the same hospital where he had been first treated, and by the same doctors). If, following the recommendations of neopositivists, the day of the week, the month, or the year in which the operation was performed is taken into account, the following generalisation might be made: 'If a person is operated on on Wednesday he will recover, but if on Friday, he will die'; or 'if a person is operated on in May, he will recover, but if in February he will die', and so on.

Medicine cannot, of course, employ such irrelevant generalisations (including irrelevant initial inductive generalisations) either in practice or to enrich its experience. If science had taken generalisations of that kind seriously, it would not have developed and concentrated experience, since it would have wiped out the difference between the essential and inessential, the chance and the necessary. Medicine, when encountering such facts (in the one case recovery, and in the second death), has to discover the conditions in which operations lead to positive results, and disclose the causes when they have a fatal outcome. Only then will it be able to create truly scientific, relevant generalisations that can be successfully applied in practice.

It follows that *a necessary condition of a relevant generalisation is non-admission of parameters that have no bearing on the essence of the matter into the corresponding descriptions of experimental dependences. That is the first methodological rule.*

Relevant initial inductive generalisations sometimes cannot be created at all if the induction is made from a simple observation only, without resorting to an experiment or without searching for a causal explanation. If, for example, Galileo had limited himself to a simple stating of the observed facts in natural conditions when he was studying the patterns of the free fall of a body to the ground, he would never have formulated the law of the free fall of bodies in a vacuum. It was because he resorted to experiments that eliminated the causes of unevenness in the fall of bodies, and built hypotheses about the causes of their uneven fall, and tested them by a hypothetical-deductive method, that he

succeeded in formulating a generalisation that is a law of nature. For that he resorted to explanation of the observed movements of bodies from the Earth (for example, the rise of smoke from a burning fire), and to explanation of other phenomena, and also to construction of corresponding idealising assumptions.

The question arises *whether genuine dynamic laws of nature can be distinguished from chance (accidental) generalisations by formal means*, i.e. in the simplest cases, propositions of the type of 'all liquids are elastic', 'all metals are electrically conductive', from statements of the form of 'all my friends are blonds', 'all my friend's cats are black'.

Bertrand Russell, considering similar matters, once suggested writing laws of nature in the form $\forall x (A(x) \supset B(x))$ ³⁵ (for example, 'For any object x it is true that it will be electrically conductive if it is a metal'). But it turned out that this form of notation can also be used for accidental general propositions: 'for any object x it is true that he will be blond if he is my friend'. Laws of nature and accidental statements written in the form $\forall x (A(x) \supset B(x))$, do not differ from one another either in truth-value or structure. Attempts to differentiate propositions about laws of nature from accidental generalisations by formal means have not yielded positive results (in any case in the context of classical logic).

Some logicians think this task is not essential for experimental sciences since we are able, in principle, to draw these distinctions at the level of analysis of concrete empirical knowledge. Solution of many other problems of logical science in general form, however, comes up against a need to distinguish between law propositions and accidental ones by purely formal means. This applies, for example, to description of the form of explanation of facts such as is based on deduction of a proposition about this fact from a law by *modus ponens*.

$$\forall x (A(x) \supset B(x)) \quad (1)$$

$$A(a) \quad (2)$$

$$B(a) \quad (3)$$

The electric conductivity of a given substance $B(a)$ (3), for instance, can be substantiated by the point that a is a metal— $A(a)$ (2), and all metals are electrically conductive $\forall x(A(x) \supset B(x))$ (1).

But $B(a)$ can be explained by the scheme adduced only when there is a law corresponding to the expression $\forall x(A(x) \supset B(x))$. If it is an accidental generalisation we get a nonsense (absurdity) rather than an explanation. Let the expression $\forall x(A(x) \supset B(x))$ be the formalisation of an accidental proposition 'All the residents in this house are short-sighted'. Then the fact that 'Fritz is short-sighted' ($B(a)$) has to be explained by his living in this house ($A(a)$),³⁶ which is absurd, although all the premisses are true, empirically checked, and the deduction correct.

In order to use this form of explanation on the general plane, therefore, we have to be able in the general case to distinguish a law from an accidental proposition by some formal criteria. The explanation can be distinguished from the pseudoexplanation when we know that the general premiss in the scheme expresses a law.

Now consider briefly the problem of *contrafactual propositions*. In language they are employed in the form 'If there were A , then there would also be B ' ('If I were not going on a business trip I would come to you'). We write them in the form $A \leftrightarrow B$, where the sign $\leftrightarrow$ is 'if ..., then ...', and A and B are any indicative propositions. The main problem in regard to this kind of proposition is to formulate the condition (criterion) of their truth. The simplest and, it would seem, most radical way may be as follows: to identify proposition $A \leftrightarrow B$ with proposition $A \supset B$, in which $\supset$ is the sign of material implication. In that case, however, the contrafactual propositions $A \leftrightarrow B$ (1), $A \leftrightarrow \sim B$ (2) both prove to be true (because of the falseness of proposition A). But this contradicts our intuition and the usage of the language; in these cases, when proposition (1) is considered true, proposition (2) is considered false.

If, for instance, a doctor says about his patient: 'If the jab had been 1 mm deeper, he'd have been a cripple all his life' ($A \leftrightarrow B$). If we trust the doctor's competence, we must consider this proposition true. Otherwise the proposition 'If

the jab had been 1 mm deeper, he'd not have been a cripple all his life' ($'A \Rightarrow \sim B'$) must be considered false. In a similar way, by identifying the contrafactual statement 'if no forces operated on the body it would move uniformly' (it being assumed that it is not at rest) with the corresponding statement 'if no forces operate on the body it will move uniformly', in which 'if ... it' is interpreted as a sign for material implication, we would have to consider the two following contrafactual propositions true: 'If no forces operated on the body, it would move uniformly' (1), and 'If no forces operated on the body, it would not move uniformly' (2). The antecedent of propositions (1) and (2) is false (in nature there are no such moving bodies, on which no forces operate); yet in accordance with the tabular definition of material implication such propositions must be considered true, but proposition (1) is true, and proposition (2) is false.

There have been various attempts to solve this problem of contrafactual propositions. One of them suggests considering the contrafactual proposition $A \Rightarrow B$ true if, and only if, A and B are interpreted as indicative propositions, and B is deduced from A and certain supplementary relevant conditions and laws G . So it is indicated that G must be a law and not an accidental proposition, which means that a general solution of the problem of contrafactual propositions must be preceded by solution of the problem of distinguishing a law from a chance generalisation by logical means.

The distinction between laws and chance generalisations is also important in the theory of induction in order to confirm inductive hypotheses. Goodman points out that this criterion of differentiation must be formulated within a qualitative theory of confirmation or within a quantitative theory of degree of confirmation, or in the context of a theory of the confirmation of statistical hypotheses.

A simple hypothesis of the general character of $\forall x (A(x) \supset \supset B(x))$ is not considered exhausted if all objects that meet the condition $A(x)$ have not been checked during its formulation. Conditions of that kind are important for forecasting. If, for a certain hypothesis, we have grounds for affirming that any object that satisfies $A(x)$ will satisfy $B(x)$ (that our proposition apparently expresses a law), then it must be

checked, employing objects with property *A* that have not been tested. Assume that we are formulating a proposition 'this illness always follows a course with a normal temperature'. This is a certain unexhausted inductive hypothesis of a general character. We consider it an empirical law. In order to confirm it we must continue our observations of the illness. We do not make such observations of new, as yet unverified cases, when it is a matter of chance generalisations of the type of 'All my friend's cats are black'. It is senseless to carry out observations on cats that are outside the limits of the experience generalised by our proposition.

In order to decide in theory when to make observations of that kind, and when not to, when to engage in confirmation of a hypothesis and when not to, we have to know how to distinguish a generalisation of the law type from chance generalisations.

Attempts to distinguish a law from a chance generalisation by purely formal means (and consequently to solve problems of the kind of explanation mentioned, and of contrafactual propositions, of confirmation of inductive hypotheses of universality, and other problems of the logic of scientific knowledge) have not been crowned with success.

The failure of neopositivists' attempts to create a logic of science was due in the first place to the fallaciousness of their philosophical maxims and also to the inadequate expressive possibilities of classical mathematical logic (above all in connection with its employment of material implication). The development of this logic had been oriented mainly on creating a mathematical theory of proof and not on resolving the logical, methodological problems of the natural sciences.

A number of logicians, both Soviet and Western, now link solution of the problem of differentiating law-governed generalisations from chance ones with the development of so-called relevant logic, the beginnings of which were laid by Ackermann in his article *Begründung einer strengen Implikation* (1956).³⁷ He posed the problem of an explication of logical investigation corresponding to 'strict' implication as a link between statements by content. In other words he posed the problem of the explication of logical investigation and conditional connection corresponding to our intuition,

based on their use in experimental sciences.

Ackermann's posing of the question was a reaction to the interpretation of logical investigation that had been peculiar to classical logic, which regarded it as a relation between the truth-values of statements. In relevant logic intensional implication is freed of the 'paradoxes',³⁸ peculiar to material implication.

I shall now employ the expression $\forall x(A(x) \rightarrow B(x))$, in which $\rightarrow$ signifies implication in the relevant sense, instead of the writing of a law, proposed by Bertrand Russell, as $\forall x(A(x) \supset B(x))$ (in which $\supset$ is material implication). When such an implication is used propositions that prove true either by virtue of the falseness of the antecedent or the truth of the consequent, lose their point. In my view employment of relevant logic so as to distinguish a law from a chance generalisation makes it possible to resolve several difficulties connected with this problem, but it is impossible to solve this problem fully by purely formal means.

The notations of $\forall x(A(x) \supset B(x))$ and $\forall x(A(x) \rightarrow B(x))$ are those not only of a law but also of a general proposition in general that equally expresses both a law and a chance (accidental) generalisation. Let us therefore try, by formal means, to divide the notation of $\forall x(A(x) \rightarrow B(x))$ into two: one part must express the law and the other chance generalisations. For that purpose we must remember what distinguishes a law from chance generalisations at the level of content. In my view they are distinguished by the following features (I have in mind only the laws of natural science, in which abstract-quantitative time is employed, and not historical time (see Chap. III, § 4)). There are laws of this type when $B(x)$ occurs for all x given $A(x)$. This dependence exists in any historical time (both now and in Plato's day, and in the future) and in any place (at any rate in terrestrial space), and in certain additional conditions.

Allowing for that, the formal writing of laws of this type will be as follows:

$$\forall x \forall t \forall R \exists I (A(x, t, R, I) \rightarrow B(x, t, R, I)). \quad (1)$$

So the law 'water boils at 100°C' holds for any (historical) time and for any space (in any case terrestrial) in certain con-

ditions (normal pressure, water without impurities, etc.). Sometimes, for example, in laws 'All metals are electrically conductive', $f = m \cdot a$, the conditions I are not recorded.

Chance generalisations are always confined to a fixed interval of time and place in space. Their formal writing will have the following form:

$$\forall x \exists t \exists R \exists I (A(x, t, R, I) \rightarrow B(x, t, R, I)). \quad (2)$$

Such, for example, are the chance generalisations 'All my friend's cats are black' and 'All the residents in that house like football'. But new difficulties arise with this. Let us consider two of them.

It would seem that we can distinguish laws from chance generalisations by the formal means of relevant logic. But formula (2) is a consequence of formula (1). That leads to new, in my opinion no less difficult, problems; each chance generalisation can be considered the consequence of a law, which generally speaking is absurd. The point is that, for example, the statements 'copper is an electric conductor', 'there are metals that are electrically conductive' can be deduced as corollaries from the law 'All metals are electrically conductive' (adding to that certain premisses and assumptions). But these corollaries do not function as chance generalisations; in them it is a matter of certain elements of the class of metals, in which the property 'electric conductivity' is *of necessity* inherent.

Laws of type (1) and chance generalisations of type (2), can be written, of course, in the language of classical logic by replacing ' $\rightarrow$ ' by the sign of material implication ' $\supset$ '. Then a curious situation arises. Writing the law by the means of classical logic leads to paradoxes of material implications, and does not guarantee sense connections between the antecedent and consequent, but does, on the other hand, help formulate strict criteria of the truth of a law (the proposition that records it is true when the antecedent is false or when the consequent is true). Writing a law by the means of relevant logic ensures a sense connection between antecedent and consequent, but does not allow us to formulate criteria by which the proposition expressing the law is true. This means that the problem of distinguishing a law from

chance generalisations is not just one of finding different recordings for them in one language or another.

Zhambaldagbaev has tried to resolve this problem of explanation on the basis of relevant logic.³⁹ The model of deductive-nomological explanation constructed by him from relevant logic serves the purposes of dealing with the problem of the sense connection between the premisses of the explanans (that by means of which the explanation is made) and of the explanandum (that which is being explained), i.e. the problem that it is impossible to solve by classical logic. But it became clear that this model did not reflect at least two special features of meaningful explanation, namely that the explanans must be maximally informative in relation to the explanandum, and that the former must explain the latter as fully as possible.

Thus the application of mathematical logic to empirical knowledge (to the determination of scientific laws, empirical explanation, etc.) is always associated with surmounting considerable difficulties. That largely applies to the most complicated, dialectical sphere of reality studied by the humanities (see Chap. III, § 4). The fact is that the apparatus of mathematical logic (having in mind classical logic) was built up under the influence of the requirements and needs of mathematics, and primarily in order to develop its foundations. Since mathematics is the domain of abstract and idealised objects and the relations between them, and differs essentially from the domain of real objects and their relations studied by the experimental sciences, difficulties arise when the apparatus of mathematical logic is applied both to the humanitarian sciences and to the natural sciences. The apparatus is employed in them in less strict form than in mathematics; furthermore, resort is made, so that it can be used, to corresponding reconstruction of empirical (including humanitarian) knowledge.

The difference between the domains of objects studied by mathematical and experimental sciences is expressed in our having to deal, in the latter, with substantiation of the truth of scientific propositions that are usually of a hypothetical character (Engels pointed this out many times), while in mathematics the truth of the propositions is not simply substan-

tiated but is proved. They are considered truths in a certain strict sense, as soon as we agree on the appropriate definitions. In the experimental sciences, however, facts of experience are the starting points of the constructs and the definitions introduced into the theory must agree with them. The basis for testing the propositions formulated is facts of experience, and not definitions and logic. The construction of empirical theories is not so strict since experience controls them, while mathematical constructs have another basis for their self-control. Since the empirical sciences are based primarily on observation and experiment (in spite of their 'theoretical load'), there is no long, complicated argument in them, as is specific for mathematical knowledge. Finally, empirical knowledge deals with more complicated real objects, dialectical in their nature, than the abstract objects studied by mathematics.

Many logicians try to solve the difficulties associated with applying the apparatus of mathematical logic to empirical sciences by developing the logic itself, by creating logics more adapted to analysing natural and social sciences. Relevant logic, as I have remarked, is one such; another is Zadeh's logic of fuzzy sets.

In classical logic an element x may or may not belong to a set A , and the relevant proposition in that case acquires one of two meanings, viz., true (1) or false (0). In the logic of fuzzy sets it is assumed that the boundaries of the sets are not definite. Elements x belong to a fuzzy set A only to a *certain extent*. Starting from the idea of a fuzzy set, i.e. one with indeterminate boundaries, an apparatus of the logic of fuzzy sets has been built up.

...Much, perhaps most of human cognition and interaction with the outside world (Zadeh wrote) involves constructs which are not sets in the classical sense, but rather 'fuzzy sets' (or subsets), that is, classes with unsharp boundaries in which the transition from membership to non-membership is gradual rather than abrupt.⁴⁰

In Zadeh's view the logic of common sense is not classical two-value (ambiguous) or even multiple value logic, but that of fuzzy sets. The machines being designed with artificial intelligence, he wrote, are not capable of performing

such tasks as recognising speech, translating languages, understanding the main point, abstracting and generalising, taking a decision in conditions of indeterminacy, and many other tasks formulated in fuzzy terms. The difference between natural intelligence and artificial

lies in the ability which present-day digital computers do not possess—to think and reason in imprecise, non-quantitative, fuzzy terms. It is this ability that makes it possible for humans to decipher sloppy handwriting, understand distorted speech and focus on that information which is relevant to a decision. And it is the lack of this ability that makes even the most sophisticated large-scale computers incapable of communicating with humans in natural—rather than artificially-constructed—languages.⁴¹

Engels drew attention to this fuzziness of the boundaries between objects, noting that it was impossible, as a rule, to draw absolutely hard and fast lines between them.⁴² The dialectical nature of reality finds its expression in this.

Zadeh considers it a very important task

to develop novel methods of treating fuzziness in a systematic—but not necessarily quantitative manner. Such methods could open many new frontiers in psychology, sociology, political sciences, philosophy, physiology, economics, linguistics, operations research, management science, and other fields.⁴³

The concept of type, important for social science, can be introduced through the concept of a fuzzy set. The former exists in social science together with the concept of set (class), and sometimes supersedes it.

In the history of science and philosophy the concept of type was developed, among others, by Goethe, Dilthey, Weber, Gempel and Oppenheim. Goethe, for instance, concluded, when working on comparative anatomy, that all mammals, as a definite set (class) of animals, had one and the same type of skeletal structure, and wrote that the concept as such already showed us how to find this type, and that experience should teach us which parts were common to all animals and in what they differed in different animals. Then, by abstraction, they could be put into order and a general image constructed.⁴⁴ It was a matter here of putting mammals in order by their type of skeletal structure, which was possible not only by singling out what was common

and identical in the structure of the skeleton but also by allowing for certain of their differences. We encounter types again at the level of biology, but even more in the social, cultural world. The concept of type, in contrast to set (class), has been extremely indeterminate until now. It could be made precise in the following way. In classical logic the element x does or does not belong to set A . The characteristic function of an element's belonging to a class is understood in only two meanings: 1, when x really belongs to A , and 0, when it does not. That cannot be said in regard to fuzzy sets. When we say that element x belongs to $\tilde{A}$ ($\tilde{A}$ being a fuzzy set), it means that x belongs to A only to a certain extent. The expression

$$\tilde{A} = [(x_1/0.2), (x_2/0), (x_3/0.3), (x_4/1), (x_5/0.8)] \quad (1)$$

means that the elements x_1, x_2, x_3, x_4, x_5 , taken from a set E broader than $\tilde{A}$ (which includes the latter as a regular subset), belong in different degrees to fuzzy set $\tilde{A}$. Thus x_1 belongs to it to an extent 0.2, x_2 to an extent 0 (i.e. does not belong to it); x_3 to an extent 0.3; x_4 to an extent 1 (i.e. belongs to it unconditionally), x_5 to an extent 0.8.

Let it be a matter of people (forming the set E that I am discussing). Fuzzy set $\tilde{A}$ is a set of tall people. Then my expression (1) can read as follows: person x_1 from the set of people (E) is tall only with a 0.2 extent of membership of fuzzy set $\tilde{A}$; person x_2 is not tall at all; person x_3 is tall only to a 0.3 extent; x_4 is an undoubtedly tall person; and x_5 has a tallness index of 0.8. Elements belonging to $\tilde{A}$ with an index of 0 are excluded from A .

Thus, if we are able to single out elements in set E that belong unconditionally to fuzzy set $\tilde{A}$ (i.e. with an index of 1), and then clarify that other elements of E have this property only to a certain extent (and we can arrange some elements of E according to this characteristic), fuzzy set $\tilde{A}$ will be a type.

Thus the fuzzy set of tall people is a type since, by starting from practical, rational considerations, we can unconditionally distinguish tall people and arrange the rest according to a height index to the boundary of unconditionally short people (the index of their belonging to set $\tilde{A}$ = zero).

The concept of the set of good decisions taken by eminent

statesmen ($\bar{A}$) is a type in relation to all the decisions taken by them (E). We can single out from the number of all decisions those that are unconditionally good-models. The other decisions can apparently be evaluated as having a membership index lying between unity and zero.

The concept of countries that belong to one socio-economic formation or another is a concept of type. Greece and Rome, for instance, of a certain period, can be unconditionally classed as belonging to the slave-owning formation; they are models in that respect. Other states and countries could be included in this formation only to a certain extent.

The concept of countries belonging to the capitalist formation in the first decades of the nineteenth century is one of type. Marx and Engels took only Great Britain as the classical country of capitalism. Marx pointed out that the patterns of one formation or another should be studied from the classical models in which the features of one formation or another were realised in its full and most developed form. The majority of the concepts employed in social studies are concepts of that kind.

Since we are often dealing with types in the social sciences rather than with sets (classes), the principle of abstraction (see Chap. I, §§ 1, 3) does not apply to them. For classes, as we have seen, any element of some set can be a representative of the whole set, since they all become indiscernible from the angle of the definite property by which we identify the objects in the set. When we are dealing with a type, its different elements represent and characterise it in a different way. The type 'countries belonging to the slave-owning formation' is exemplified most fully and deeply, for example, by Greece and Rome, but not by other countries in which slave-owning was not manifested in its classical form. This means that *the dialectic of the individual and the universal operates in the social sciences in a much more complicated way than in the natural sciences and mathematics.*

§ 6. On the Nature of Individualisation
and Generalisation in Fiction

In order to go more deeply into my theme and achieve greater clarity, it would be useful to compare artistic generalisation with scientific. Let me first examine the special features and main regulators of art and science as forms of activity.

Scientific, theoretical activity, and its forming and functioning, are determined and governed by such epistemological controls as truth, applicability in practice (sometimes through an aggregate of other theories), and the 'logic' of the object, i.e. the appropriate patterns in the relations of objects of the surrounding world and in the trends of their development.

These controls are absent in artistic creation and its products. Truth as a reproduction of reality, in the sense of its reflection (with which we are concerned in science), is replaced by *artistic truth* in artistic works. It may be contained in fairy tales, fables, and other works of art with patently imaginary subjects, non-existent situations, personages, and events (recall Goethe's *Faust*, Dante's *Divine Comedy*, Pushkin's *Ruslan and Ludmila*, Anatole France's *Penguin Island* and *Revolt of the Angels*, in which various mythological and fantastic creatures, etc., act).

A writer constructs most artistic images, 'personages and situations from study and knowledge of the life and its contradictions, and of the characters he meets in it. Knowledge in art, moreover, whatever form it takes, 'is inseparable from the experience that permeates the whole process of artistic assimilation of the world'.⁴⁵ The personages and situations invented by an artist, even if they did not exist objectively in the form in which they are represented in artistic works, are quite real in the sense that they were drawn ultimately from real life; certain elements of the most fantastic images, for instance, are taken from life, and then freely combined and considerably modified.

But the creativity of the novelist, short-story writer, poet, and dramatist differs from scientific activity. The fundamental empirical sciences, of course, bring out the objective characteristics of reality through tables of observations and

their subsequent generalisation and application, as a result of which laws are formulated, and then a mathematical apparatus applied to them. During scientific analytical activity, on the basis of the data of an experiment, concepts, abstractions, hypotheses, and idealisations are formulated, and these serve as a means for deeper reflection; during application of the theory in practice idealisations are replaced by more direct images of objective reality.

In the fundamental empirical sciences the logic of the object determines the logic of the reflection in the last analysis. The artistic images in general lack the determinacy that is peculiar to the components of science. Artistic creation is closer in a sense to scientific-technical creation, and the practice based on it of creating new objects that did not exist before man and apart from his creativity; it is the matter of creating something new, not previously encountered. One must remember here, of course, that technical creation, and the practice founded on it, are based on the objective laws, and regular relations elicited by the fundamental sciences, while the images created by the artist are not in general strictly determined by the objective laws of nature and social life. But nature, social life, the environment, and objectively existing 'laws of beauty' determine certain facets and characteristics of an artistic work.

The close link with practice and direct utilitarianism of many scientific theories are replaced in artistic works by a capacity to evoke aesthetic feelings and experiences in the addressee, on the basis of which, and in the aggregate of which, ethical and political social ideals and values, and the educative role of an artistic work, are realised. Such are the aim and purpose of art in society, and in that lies its necessity.⁴⁶

The objective 'logic' reflected in sciences is replaced in art, as a rule, by artistic sense, which is the controller of the development of the action in artistic works (an exception in this respect, perhaps, is artistic works on historical themes).

What is meant by artistic sense?

A sharp distinction is made in logic between the terms 'meaning' and 'sense'. Meaning is what is denoted, while

sense is the means by which this denotation is expressed, represented, put across, and conceived. In its constructive-cognitive aspect, sense can be defined as the modes of describing and interpreting that which is fixed in language, and as the determination of these modes by the aims, precepts, and tasks of the subject who is carrying on a scientific investigation, which are always limited by the context of objective facts and laws. This concept of sense is based on the 'logic' of the object studied and the logic of cognition.

By *artistic sense* is meant that mode of depicting, representing, and expressing a situation in artistic, poetic language, which is determined (a) by the ideas, intentions, and ideals of the artist (which have a social nature) and by his ideological and psychological orientations, and also by the object of the depiction; and (b) by the simultaneous striving to evoke social feelings through the aesthetic in the addressee and to influence his behaviour.

The educative function of art is realised on that basis. Its fulfilment presupposes knowledge in the artist of what is depicted, knowledge of life (the situation and relations of people and their moods, etc.), and of what underlies realisation of the *cognitive function of art*.

Artistic sense in that understanding of it determines the choice of what is depicted, the interest in it, its transformations and distortions, its relation to artistic achievements of the past, and the development and interpretation of what is depicted. It underlies the forming of an artistic work as an original, unique masterpiece of artistic culture. This sense (unlike its constructive, cognitive definition) is not determined in the general case by the objective 'logic' of what is depicted. In artistic literature (belles-lettres, poetry, and fiction) such a technique is often employed as reversing the relations of the conditioner and the conditioned, which is not permissible in science.

Recall, for example, the discussion in connection with the arrival of the government inspector (in Gogol's play *The Government Inspector*)⁴⁷ about what dresses Anna Andreevna and Maria Antonovna should wear. Anna Andreevna said that she would wear her primrose gown because it went with her dark eyes. Maria doubted that her mother

had dark eyes. A row developed. The mother justified her position by the argument that she told her fortune by the Queen of Clubs. Because of the stupidity of this argument, which arose through a change of place of cause and effect, it makes the addressee smile (affects his emotions) and at the same time characterises the intellectual level and interests of the talkers.

Artistic creation is thus a form of constructive activity. It is manifested in clearest form, not requiring special argumentation, in music and architecture, for example. In fiction and painting the constructive activity is closely knit with the portrayal of life itself, or with those elements of this portrayal that are drawn in the last analysis from life, and from people's various notions. As I have said, artistic creation is like technical creation, though it is not strictly determined by objective facts and laws. It must always, however, conform to certain requirements, breach of which is connected with transition from the sphere of artistic creation to another sphere of activity, for example in games whose rules are based only on agreement.⁴⁸

The thesis formulated above only gets a quite precise sense when aesthetics fully answers the question of *what*, precisely, in a form specific to art is capable of arousing aesthetic feelings and experiences. It may be objected that not only are scientific-technical and artistic creation constructive but also the purely scientific activity by which scientific theories, proofs, hypotheses, etc., are created and 'constructed'. That is so, of course. But scientific activity is determined and limited to a much greater extent by objective circumstances than scientific-technical, inventive activity and artistic creation.

The language employed in science and art plays such a different role and has developed in such different directions that one can speak of the *language of science* and the *language of art* as phenomena different from one another.

Both have their source in the ordinary language. The language of science was formed in the first place through a terminology related to one field of science or another. Its terms were developed above all through the refinement of words and phrases of the ordinary language with allowance

for each term's corresponding to one, single object (an individual or class of objects) within a given domain of knowledge, which meant that homonyms were eliminated from the ordinary language employed for scientific purposes. Words and phrases were made precise by singling out and rigorously fixing their main lexical meanings, which became, generally speaking, independent of the context. Rules of identification were introduced for various words and phrases by means of clear definitions. These rules took the form: 'term Dfd $\equiv$ term Dfn' (in which $\equiv$ is a sign establishing the identity of these terms according to their meanings).

The development of the ordinary language toward a scientific language presupposed its transformation in the direction of greater precision and the elimination of ambiguities, subjectiveness, and emotionality, and in the direction of achieving greater adequacy of the description to the objects studied. Subsequently scientific language was enriched by artificial terms and, moreover, ideographic writing. The signs =, >, <, H₂O, etc., no longer signify phonemes, as in the natural language, but the objects themselves directly, and also their connections and relations. The introduction into the language of science of a language of formulas, and of signs for variables, that made it possible to write general dependences in simple compact form, and to introduce the concepts and apparatus of mathematics into the natural sciences, played a great role in its development. The so-called formalised languages of logic are the pinnacle of the language of science.

The language of fiction and belles-lettres evolved in another direction. Means were developed and perfected in it that could evoke emotions and protracted aesthetic experiences in the addressee, that could give the possibility of depicting the complex, dialectical world and life of the depicted subject more deeply, that could enable the author's intentions to be embodied in graphic form, and could also give constructive activity the freedom and persuasiveness without which artistic creation does not exist. The mediated nature and extraordinary variety of the relations between people, with their different characters, called for the creation of linguistic means by which they could be expressed in a practically infinite number

of variants and contexts and, moreover, allegorically in an evocative, and informationally rich form (compare the means of expression of wit, humour, irony, mockery, admiration of nature, love, etc.).

To illustrate the substantial difference between the language of science and the language of art, let me cite some examples from highly developed poetic language. Their interpretation and analysis as examples of scientific language will show the inadmissibility of identifying the two.

In order, for instance, to express the sense of the proposition 'This man is young' (1), formulated in terms of ordinary language gravitating toward the scientific, the poet, employing metaphor, can represent the idea of it as follows: 'there is not yet a grey hair in the soul of this man' (2). The method of scientific checking can be applied directly to proposition (1). There are corresponding techniques in medicine and jurisprudence; the actual age of the person is established by means of them, when, for example, his documents are untrustworthy.

If it is discovered that his age is not young, then proposition (1) will be false; in the opposite case it will be true. Proposition (2) can only be considered an adequate translation of proposition (1) in a certain sense (i.e. in an intellectual-extensional sense). But proposition (1) differs essentially from proposition (2): the latter has a colourful form capable of evoking aesthetic emotions and ideas corresponding to the metaphorical, figurative sense of the words 'a grey hair' (grey hairs usually being a sign of old age); by asserting that there is not a grey hair in the soul of a person we at the same time indirectly say that he is young.

Now let me examine proposition (2) (and also proposition (1)) as a phenomenon of a section of ordinary language in which the cognitive-practical experience of mankind is recorded (i.e. let me interpret it as a certain primitive scientific proposition). Let me try applying the methods of scientific checking (verification) to this proposition. To do so, I must obviously seek out in him an object, like a soul, on which hair grows, and to clarify whether there are grey hairs on it. If there are, then proposition (2) needs to be considered false;

otherwise it is true. But understandably the checking procedure described is inapplicable in principle to proposition (2), and therefore senseless and absurd. Such a conclusion applies to all metaphorical expressions (for example, 'This painter has still not cut his wisdom teeth', 'The cocoon of distrust has metamorphosed into the moth of fear').

Similar cases are obtained when we analyse epithets and similes. Consider such epithets as 'icy hands' and 'dead silence'. Certain characteristics in them ('icy' and 'dead') are applied to hands and silence. If the combinations are understood immediately and in a direct sense (as is always done in science), we get nonsense; people do not in general have hands consisting of ice, and silence is not dead in the sense in which a person is dead (i.e. in the sense of an absence of life). To say 'dead silence' is equivalent, from the scientific point of view, to saying 'fuzzy air', a 'salty functional', and so on. When we analyse these metaphors in artistic language, they have a profound sense. One can say 'icy hands' because ice has the attribute of cold, and the latter is exemplified by it. Use of the epithet 'icy hands' instead of 'very cold hands' is not only justified intellectually but also gives a chance of representing the property 'very cold' in the form of an evocative image. The same can be said *mutatis mutandis* of the epithet 'dead silence'.

Characteristics in the form of properties (as in my examples) are also ascribed to objects in science. But in it the word combinations 'iron bed', 'bivalent metal', etc., contain the properties 'iron' and 'bivalent' taken as belonging immediately and in a direct sense to a bed and metals respectively. I therefore find the following statement in the encyclopaedia article 'Epithet' unconvincing:

There are no grounds for attempts to draw hard and fast lines between artistic definition (i.e. epithet) and logical definition that indicates the general but not the particular properties of an object (for example, 'iron bed'), since logical definition can also have expressive significance in an artistic context.⁴⁹

In artistic works inanimate nature interests the artist only insofar as it enters and is interwoven into the life of man. The abstractions that often figure in such works are not simply

reified, i.e. treated as existing in the same sense as material single objects, but are also personified and endowed with life: for example, 'But love still lives in my sick heart', 'Bitter grief gads about the world', 'Pensiveness is her companion', 'He was consumed by envy', 'A thirst for power had gripped his whole being'. Application of the methods of scientific verification to these propositions (without their appropriate translation) leads to absurdity, as in the examples cited above.

Science and art are thus essentially different spheres of activity. 'Science and art are not interchangeable, and they simply do not dissolve into one another'.⁵¹ That, of course, does not prevent their having a reciprocal, fertilising influence.

Let us look now at the problem of individualisation in the forms of generalisation in art. Let us begin with an analysis of the individual. The problem of the means of specification, i.e. the allotting of proper names to separate objects of the field studied, is not a secondary one either in science or in practical life. We constantly have to deal, for example, with separate, single abstract objects in mathematics (numbers), with concrete bodies and particles of one kind or another in physics, chemical elements in chemistry, and so on. Sciences like geography, astronomy, and history, are inconceivable without the introduction of a vast set of proper names for separate, single objects. The basis for the specification of objects by proper names is the fixing in them of properties inherent only in them (in any case among objects of the subject field studied by the scientific discipline that it forms part of).

Such specification is sometimes limited to fixing its individualising characteristics, i.e. the creation of so-called definite descriptions, the logical theory of which was developed by Bertrand Russell. Such are the descriptions: 'x is the inventor of the wheel' (1), 'x is the longest river in Europe', 'x is the number greater than zero whose sum with itself is its square, i.e. $x \equiv x > 0 \wedge x + x = x^2$ ', and so on.

The specification can be made by other means. An important method of the specification of separate objects, for instance, of their parts in theoretical constructs and models, is

a reference grid. It individualises an object by means of co-ordinates that characterise its location in such-and-such points of space at a certain time.

A necessary condition of the normal development and functioning of public life is the specification (individualisation) of people, towns, streets, houses, rivers, mountains, etc., by giving them proper names. Each person, whose outward appearance and inner character is changing all the time, receives a name at birth that is recorded in appropriate documents. The general principle of the proper individualisation of the objects of science is formulated as follows: objects are properly individualised in various scientific descriptions insofar as they reproduce the objective characteristics of the objects existing in the field studied, for which these characteristics are specific and distinctive.

This principle is not fulfilled in general in art, with the exception of portrait painting, and landscapes that depict memorable places. Each situation in actual reality, and each of its fragments, are strictly speaking unique (the principle of individuation already formulated by Leibniz, incidentally, refers to this). It is impossible to make two identical material objects indistinguishable from each other in some absolute sense. The faces and fragments of reality depicted in the kinds of painting mentioned above, and moreover with sufficient accuracy and the requisite skill (mastery), will correspond to some extent to the scientific principle of individuation formulated above. There is an authenticity of description in them analogous to that which we encounter in science. In belles-lettres there are historical novels, memoirs and reminiscences, and descriptions of the life of famous people. In these literary genres, techniques of individualisation similar to scientific ones, are therefore often employed to distinguish persons and events.

But things are different in literature in most cases; the principle of individuation is violated in a certain sense at every step in artistic description. It is usually a matter of indifference to us whether or not a character had a prototype, and whether or not the prototype's life has been adequately described. If it were made clear, for example, that there was no prototype for Anna Karenina, our attitude to

Tolstoy's novel and appreciation of it would hardly be altered. It is therefore not a matter in belles-lettres, as a rule, of adequate, objective individualisation of events and personages that existed in life. The individuation in it relates to the world of the artistic work created by the writer (its personages, plot, circumstances, etc.), in which features of the world of reality are fixed in some form or other.

In other words, the writer individualises the personages in the artistic work itself and the world constructed in it. A writer creates a gallery of images differing from one another. The same must be said of the episodes of life, situations, and circumstances he describes. These episodes and personages are not individuated by means of definite descriptions, as in science, to which the proper names correspond, but by artistic means. A major role is played here by descriptions, by the means of artistic language, of the personage's characteristics, actions, attitude to himself and others, and to his environment, and of his moods, experiences, and thoughts, and of his life in general. The individualisation of the main characters is much richer than that of the secondary ones. Everything that is said, for example, about the main character or other personages in a novel or story can be regarded as their individualisation. It takes place throughout the work.

Certain individuated fragments of life and life situations are constructed on the basis of the individualised personages, circumstances, and events described in a work. Although the artist's description of immediate situations plays a major role in the process of artistic individuation, it is done through features carefully selected by the writer in the course of artistic thinking. An artist selects those features and details that are typical of such-and-such a personage and of such-and-such events and circumstances. This concept of the typicalness of a description is akin to that of the essential in scientific knowledge. But while the essential is found in science in an object's objective features, the typicalness of the persons, doings, and events described in an artistic work is determined first and foremost by the author's plan, the inner unity of the work (unity of form and content), and its truth to life, and the striving to achieve the maximum aesthetic

effect on the addressee. These determinations are essentially ideological and artistic, and depend on the sense of the work. The artist, if you like, employs a certain analogue in the process of individualisation of the method of ascending from the abstract to the concrete developed by Marx in relation to the process of scientific knowledge.

In some partial cases a situation individuated and described in an artistic work may be drawn wholly from life; then the choice of features is associated with the seeking out of the situation itself and with the singling out of the most persuasive elements drawn from the reality described. In other cases the living facts and events only give an impulse to the description of the situation in the artistic work, but are then supplemented by imaginary facts in accordance with ideological, artistic determinations based on the plan and sense of the work. Artistic individuation is directed and governed by them, too, when the writer is dealing with wholly fictitious situations (in fairy tales, fantastic novels, etc.).

The artist's personal world, the clashes experienced by him, his dreams of what still does not actually exist, and his hopes, losses, and disillusionment, play a great role in the choice, completion, and creation of an artistic situation. André Maurois wrote in this connection in his book about Balzac:

He gave himself what life had refused him; from Marsay he took the beauty and power; from Rastignac the rich marriage, the great career; from d'Arthez, innocence. Or otherwise, by a magic process as old as humanity, he freed himself from his checks by attributing them to a personage that he crushed. Thus Rubempré delivered him from his difficult youth; Birotteau from the sinking printing business; Nathan from his writer's agonies.⁵¹

Halldór Laxness, for example, justly remarked that the images of proud, courageous, noble people described in Icelandic literature of the thirteenth and fourteenth centuries were created in a period of a far-reaching decline in the history of the Icelandic people, when Icelanders were not, as was known, proud, or courageous, or noble, when they were conquered by foreign invaders, and when the moral fibre of the nation was undermined. At that time, he wrote, when the national values of Iceland faced destruction, the heroes

of the sagas, and the heroic situations described in them, embodied the dream of greatness of a conquered and humiliated people. Belief in the reality of those images proved stronger than any other faith in Iceland.⁵²

Let us note three, in my opinion essential, factors in what I have said above.

(a) The artist, in trying to lend reality to the situation described (even if it is fictitious from beginning to end), gives the characters names suitable to the given situation, and also gives names to towns, rivers, and streets, and describes episodes with the naturalness proper to real events. That helps the addressee to materialise the content of the work in images of actual life during his reading of the work, and to adduce features of reality to what clearly lacks it. This psychic Gestalt-switching functions as a condition of the beginning of interest in what is described, and in the fate of the characters, which is a necessary factor in the development of emotion.

(b) The artist's creative, constructive activity, including individualisation, is similar in certain respects, as I have already said, to scientific-technical activity. The latter achieves its purpose only when the technical devices planned and created by it correspond to certain pragmatic criteria; when they are employed for the good of man and society as a whole. A new farm machine, for example, created to harvest a certain crop, is good if it meets its purpose, if it lightens and economises man's labour, which means that its constructors have studied and correctly reflected and embodied in their creation a certain sum total of objective patterns described by the fundamental sciences. It does not follow from that, however, that other machines, not resembling this one, could not have been created in order to realise the same ends. They might differ from it essentially in many parameters, but the constructor activity must in any case be based on objective laws.

Even more significant, essentially unlimited creative opportunities are opened to the artist, since they are governed in principle not by the objective laws of science, but *by the ideological and artistic determinations and senses mentioned above*. That also applies to the process of artistic individ-

ualisation. We can say that an artistic work succeeds when it satisfies high ideological-artistic criteria, which means that it is capable of having an effect on the mind and outlook of the addressee, and that it has educational significance.

(c) As I have already mentioned, there are artistic works that are linked in varying degree with life. But even those that are far from reality, and deal with imaginary situations and images, are oriented on real life. Without knowledge and study of life it is impossible to attain artistic truth. The processes of individualisation also presuppose this orienter.

There is a whole system of individualisation of fiction.

(1) There is the so-called *primary-formal individualisation*, by which is meant the specification of personages through proper names, age, relationships with other personages of the work (family, ideological, love, service, neighbourly, etc.).

Formal individualisation makes it possible, within the limits of a work, to specificate the characters and to differentiate them from one another. But that is not just characteristic of artistic creation; it is also employed in science, and in various spheres of material activity. Before we carry out an investigation, for example, we have to individuate, and identify, the subject (object) interesting us from other similar objects. Individualisation is a formal, necessary condition of orientation in any sphere of activity and should protect us from false identifications.⁵³

(2) What is known as *artistic-content individualisation* of a work as a whole is encountered in artistic creation. With a broader approach to the essence of the matter, we can pose the question of the individualisation of a work within the context of artistic creation in general, and within the framework of the most varied artistic works written by different authors.

If we single out the relations described above between the characters in a certain general form, and fix the evolution of their lives and relationships schematically, without allowing for who the work is written by, what it is called, and what are the characters' names, we get a certain scheme that no longer individualises the work as a whole and does not make it and its dramatis personae unique. This scheme

will be a plot or story that can be realised in a host of other artistic works.⁵⁴

The artistic content of a work as a whole is individuated in general for artistic creation only when its marked scheme (plot) is supplemented by clear, colourful, evocative ideas, reserved for the names of some of the characters, and the description not only of their actions but also of their motives and inner emotions, treated in the evolution of the characters, allowing for place and time.⁵⁵ I call such individuation of a work as a whole artistic-content individualisation.

It is the business of the artist to evoke in an adequately prepared addressee such a combination of graphic images as would individualise the situations and personages described, create an illusion of actual life and arouse aesthetic feelings in him. The difference in the names of the characters in separate works is supplemented by their differentiation from one another in their graphic-evocative characteristics and in their activity and psychic features. Both develop in time and function in this form as individualising characteristics. The secondary and minor characters understandably do not usually get as rich and concrete individualisation as the primary ones. The individualisation embraces the whole process of the artist's creativity in making a work of art. Artistic-content individualisation is not only employed by representatives of the realist trend but also, for example, by writers of the romantic trend. Belinsky wrote of Walter Scott's novels that each person in them 'exists by himself for us; you see him before you in all his stature, in all his characteristic features, and will never forget him.'⁵⁶

(3) An artist often individualises personages, while at the same time identifying them (at least in some of their characteristics) with members of some social group, making them spokesmen of the interests of given groups. Sometimes, in the course of individualising, a writer endows his personages with features encountered in members of various social groups. In both cases this is often spoken of as typification of the personages, and may be a social-group typification or one common to humanity. *I call this individualisation typifying.*

(4) There is an individualisation in artistic creation that

is associated with a specification of a personage that brings out and distinguishes him from members of the social group (class, estate, tribe, party, family, sympathisers, collective in which he works, etc.). It may characterise a *dramatis personae* of a work as a fighter for progress who opposes the old, outlived, dark, and reactionary, as a hero, or as a clearly expressed negative phenomenon against the background of his environment. This individualisation can be called *anti-typifying*. The positive heroes of works often separate themselves from their social environment, waging a struggle against obsolescent morals, principles of life, the views and traditions dominant in society, injustice, tyranny, corruption, etc. Sometimes a tragic end befalls them. Examples of such positive characters are Katerina in Alexander Ostrovsky's play *The Storm*, Helen in Turgenev's novel *On the Eve*, Fabrizio in Stendhal's novel *The Charterhouse of Parma*, Hamlet in Shakespeare's tragedy, Rob Roy in Walter Scott's novel, and so on. But there are works in which the writers do not resort to individualisation of that kind. Such, for example, are Pushkin's *Tales of Belkin* and verse novel *Eugene Onegin*, Chekhov's plays, etc.

Now let us examine the process of *artistic generalisation*, by which I understand (A) the constructing and description by artistic means of an individual such as is a representative of the general, i.e. of a whole set of objects (social classes, strata, estates, groups of people with similar features of character; they are also close in enthusiasms, behaviour, fate, etc.); (b) extension and extrapolation of the general in the sense mentioned above beyond the limits of the artistic work to phenomena of life itself.

Artistic generalisation in the first sense is realised in works in the form of typification of personages and *events*, and *in the form of their idealisation (typologisation)*. Typification is also nothing other than the representation of the general by artistic means through the individual, and, moreover, so that the object of the construction and description is only individual. The features of the general are merged organically with the individual, immersed in it, but 'shining' through the individual, unique, inimitable, and original. Belinsky, when characterising the typical, wrote:

There is another law in artistic work: it is necessary for a person who is the expression of a whole, special world of persons, to be at the same time a unique person, whole and individual.⁵⁷ ... Only on this condition, only through reconciliation of these opposites can it also be a typical person in the sense in which we call Othello and Major Kovalev typical persons.⁵⁸

A characteristic feature of typifying is full merging of the universal and typical with the individual, which appears in the work as if taken whole from real life.

Each of Shakespeare's persons (Belinsky wrote) is a living image, with nothing abstract in him, *as if* (my italics—D.G.) taken as a whole and without any modifications and alterations from everyday reality.⁵⁹

The typical features of certain classes, estates, and social strata can be embodied in individualised, unique form (compare the main personages of Gorky's plays and novels *The Petty Bourgeois*, *Vassa Zheleznova*, *Dostigayev and Others*, *The Artamonovs*, *The Life of Klim Samgin*). The features of lone rebels belonging to various social groups (Dubrovsky, Raskolnikov) are also depicted in the same form. In other works types are created that can be met at various epochs and in various countries (compare Don Quixote, Faust, Hamlet).

In this connection it is not out of place to recall that Chernyshevsky considered Khlestakov extraordinarily original, but nevertheless believed that there were very few people in whom there was not a sign of Khlestakov. Gorky, considering typification in fiction, wrote:

The characteristic feats of many heroes are 'abstracted', singled out—and then these features are 'concretised', generalised in the form of one hero, say Hercules, or the Ryazan peasant Ilya Muromets; the features most natural in each merchant, nobleman, peasant are singled out and generalised in the person of one merchant, nobleman, or peasant, and so we get a 'literary type'.⁶⁰

Here an important methodological proposition is made about the organic link of the individual and the universal, the single and the many.

The process of typifying also admits of exaggeration, exceptionality, and even imaginary unreal situations, in which

the characters act (compare Balzac's *The Wild Ass's Skin*, Shakespeare's *Hamlet*, Gogol's *Viy*). These techniques make it possible to disclose characters and living truth that stimulate the interest of the addressee. When Balzac published his novel *La Fille aux yeux d'or* (*The Girl with Golden Eyes*) with an improbable plot, he was asked whether the episode described by him had actually happened. He replied: 'The episode is true'.⁶¹ Belinsky wrote:

Hamlet is a whole separate world of real life and you see how simple, ordinary, and natural this world is for all its *extraordinariness* (my italics—D.G.) and loftiness.⁶²

In realistic works (but not just in them) certain parameters of a personage's character are usually generalised. Pushkin noted that aspect of it: 'In Molière *The Miser* is mean and only mean; in Shakespeare Shylock is tight-fisted, shrewd, vindictive, philoprogenitive, and witty.'⁶³ In multiparametric generalisations some features are basic and others secondary, which is governed by the main idea of the work. I have already remarked that generalisations are realised in artistic works not only in the sense of typification, but also of idealisation (*typologisation*). Belinsky identified 'idealisation' in essence with typification.

In other words 'to idealise reality' means to express the general and infinite in a partial and finite phenomenon, without copying any chance phenomena, but creating *typical* images, that owe their typical nature to the general idea expressed in them...⁶⁴

Consequently, to idealise reality does not so much mean to embellish it as to show it, like a divine idea, that carries in its recesses the creative force of its realisation from non-being into a living phenomenon.⁶⁵

That identification of typification and artistic idealisation can be considered legitimate.

The existence of two types of generalisation, distinct from one another, has already been noted in aesthetic literature.⁶⁶ Dneprov suggests adopting the term 'idealisation' to denote generalisation as distinct from typification. Gulyga recommends the term 'typologisation' for the same purpose. Both terms have their good and bad points. I shall use the term 'artistic idealisation' for this purpose, having in mind that the term 'idealisation' is intuitively associated with words like 'ideal',

and 'idea' of the ordinary language, which are less polysemantic than the word 'type', with which the term 'typologisation' is associated.

By *artistic idealisation* I mean the modes of generalisation and organisation of artistic forms of work through approximating them to certain patterns by eliminating everything from them that does not conform to the writer's ideas, aspirations, and ideals, and at the same time by adding to them what does correspond to them. In works that employ methods of artistic idealisation the general, and sometimes the conventional, are brought to the fore, while the individual is schematised. The psychological description of individual characters in connection with social conditions and circumstances gives way largely to description of the actions and deeds of the personages, and to complication of the story line and event content of the work.

Neither typification nor artistic idealisation exists in pure form. The point concerns the preferential use of these methods in one literary work or another. The fulness and individualised nature of the characters is an essential feature of typification. Pushkin noted the existence of such typified images in Shakespeare. Molière described the Miser by means of artistic idealisation. 'Does this mean,' Gulyga asks, 'that Molière was a poorer comedy writer than Shakespeare?' He answers: 'One can argue about that'.⁶⁷

Examples of typical images are Pechorin, Julien Sorel, Raskolnikov, Natasha Rostova, Pierre Bezukhov, Anna Karenina, and Grigory Melekhov. They create an impression of the almost boundless fulness and wealth of the characteristics embodied in them, although some of them are brought to the fore by the writer. Images like Herakles, Prometheus, Manfred, El Cid, Rakhmetov, the Count of Monte Cristo, D'Artagnan, Mephistopheles, the Demon, Sherlock Holmes, the heroes of Jules Verne's novels, the characters of modern science fiction and detective stories, the personages in the plays of Mayakovsky and Brecht, certain characters in Bulgakov's novel *The Master and Margarita*, etc., are created by artistic idealisation.

The method of artistic idealisation is broadly used in fairy tales. In them are depicted

the cruelty and insolence of the rich and powerful, yet in the final count, contrary to what happens in harsh reality, it leads by the magic of fantasy to the triumph of the poor and down-trodden. It gives a momentary rest from the victory of evil, and momentary satisfaction and a burst of happiness—Johnny Dumb-cluck in a gleaming mansion with a bride of indescribable beauty.⁶⁸

In Kafka's novels, created with employment of the method of artistic idealisation, the reader encounters a blank map as it were, and

each one, in accordance with his interests, knowledge, capacities, and experience, has to fill in the schematic picture presented by the author.... Understatement and compression of thought are combined here with a directly opposite technique, viz., an abundance of statements and compression of material. Poorly organised life, and bureaucratic morals grow here to a prodigious scale and are converted into a real phantasmagoria.⁶⁹

Dneprov notes in his book *Problems of Realism* that we also meet typification and artistic idealisation in other forms of art. Greek sculptural portraits of young Athenians of the fifth century B.C. were not portraits bearing their real historical and national characteristics. They were not realistic, typifying heroes. The artists strove to create ideal, heroic images that still, after thousands of years, continue to move the viewer and evoke his admiration. It was not by chance, therefore, that Marx compared the sombre typifying realism of Rembrandt to the idealised creations of Raphael.

The methods of artistic idealisation find expression in lyrics, music, ballet, and the circus.

'It was also of great importance that music or lyric poetry, taken as a whole, could not help by its very nature, having a positive image as its general content.'⁷⁰

In these literary and musical genres the individual-generalising, socio-typical, and realistic are met much less often.

The Hungarian philosopher György Lukács once mistakenly suggested that where there is no realism and typification the artistic image disintegrates and art becomes degraded in general. In fact, typifying and artistically idealising methods of creation coexist (sometimes even within one and the same work). These methods are united by the single, conventional

nature of art. 'There is no absolute, unconventional art (that does not distort reality). Only in some cases does art conceal its conventionality, in others it flaunts it.'⁷¹

The work of art created by an artist (if it is truly artistic) is generalised by the vast mass of addressees in the sense that they begin for a time to perceive what is depicted in the work as life without embellishment, as genuine reality. There is an extrapolation of the narrative created by the artist, unique and sometimes partly imaginary, to a set of real life situations that appear in the consciousness of various addressees as similar but, generally speaking, not wholly identical. Readers and viewers believe the artistic information to different degrees, but at the same time realise that it is an artistic work that is not life itself.

¹ As we showed in Chap. I, § 7, this extension, when properly interpreted, can be treated as a generalisation of the corresponding statement.

² See I.V. Kuznetsov. Mathematical hypothesis, in *Filosofskaya entsiklopedia*, Vol. 3, pp. 336-338.

³ *Ibid.*, p. 337

⁴ *Ibid.*, p. 338.

⁵ See David Hume. *An Enquiry Concerning Human Understanding* (London, Printed for A. Millar, 1853), p. 44.

⁶ *Ibid.*, p. 33.

⁷ *Ibid.*, p. 46.

⁸ *Ibid.*

⁹ *Ibid.*, p. 36.

¹⁰ *Ibid.*, p. 37.

¹¹ *Ibid.*, p. 34.

¹² *Ibid.*, p. 31.

¹³ *Ibid.*, p. 43.

¹⁴ *Ibid.*, p. 35.

¹⁵ See G. Polya. *How to Solve It* (Princeton U.P., Princeton, N.J., 1946), p. 98.

¹⁶ *Ibid.*

¹⁷ See Karl Marx. *Capital* Translated by S. Moore and A. Aveling (Progress Publishers, Moscow, 1978), pp. 54-75.

¹⁸ It is a matter here of the operation in thought with formed concepts rather than of the historical movement of knowledge.

¹⁹ A.L. Subbotin. Idealisation as a Means of Scientific Understanding. In T.V. Tavanets (Ed.). *Problemy logiki nauchnogo poznaniya* (Nauka, Moscow, 1964), p. 366.

²⁰ *Ibid.*, p. 367.

- ²¹ *Ibid.*, p. 366.
- ²² Karl Marx. *Op. cit.*, p. 163.
- ²³ *Ibid.*
- ²⁴ V.I. Lenin. Once More on the Theory of Realisation. *Collected Works*, Vol. 4 (Progress Publishers, Moscow, 1977), pp 74-93.
- ²⁵ *Ibid.*, pp. 76-77.
- ²⁶ See Lenin's letter to A.N. Potresov in *Collected Works*, Vol. 34 (Progress Publishers, Moscow, 1977), p. 34.
- ²⁷ V.I. Lenin. *Art. cit.*, p. 86.
- ²⁸ *Ibid.*
- ²⁹ *Idem.* Our Programme. *Collected Works*, Vol. 4, pp. 211-212.
- ³⁰ *Idem.* Once More on the Theory of Realisation, *Op. cit.*, p. 87.
- ³¹ *Idem.* Imperialism, the Highest Stage of Capitalism. *Collected Works*, Vol. 22 (Progress Publishers, Moscow, 1974), p 265.
- ³² V.Kh. Vasilenko (Ed.) *Propedevtika vnutrennikh boleznei* (Introduction to Internal Diseases), Meditsina, Moscow, 1974, p 12.
- ³³ V.I. Lenin. Report on the Party Programme to the Eighth Congress of the RCP(B), March 19, 1919. *Collected Works*, Vol. 29 (Progress Publishers, Moscow, 1977), p 168.
- ³⁴ I have in mind here primarily initial inductive generalisations.
- ³⁵ The quantifiers of generality $\forall x$ in expressions like $\forall xP(x)$ ('for any x it is true that it has property P ') have a different sense in natural science and in mathematics. In natural science the expression $\forall xP(x)$ presupposes that each x has property P in certain conditions. In mathematics there are no such conditions varying for each case. In it propositions like $\forall xP(x)$ are taken as conditions or are proved (for example, by means of perfect mathematical induction). In mathematics, moreover, time is abstracted during formulation of its dependences.
- ³⁶ Stegmüller. *Wissenschaftliche Erklärung und Begründung* (Springer Verlag, Berlin, 1969), p 274.
- ³⁷ Ackermann. Begründung einer strengen Implikation. *Journal of Symbolic Logic*, 1956, 21, 2: 113-138.
- ³⁸ See E.K. Voishvillo. Logical investigation and implication (semantic analysis). In: M.V. Popovich (Ed.). *Aktual'nye problemy logiki i metodologii nauki* (Naukova dumka, Kiev, 1980), pp. 173-193. By paradoxes of material implication is meant that it follows from its tabular definition, that anything you like (both true and false) follows from a 'lie', and truth follows from anything you like.
- ³⁹ See N. Ts. Zhambaldagbaev. On deductive-nomological models of explanation. *Filosofskie nauki*, 1982, 1: 142-145.
- ⁴⁰ L.A. Zadeh. Foreword to A. Kaufman. *Introduction à la théorie des sous-ensembles flous (Fuzzy Sets Theory)*, (Moscow, Paris, 1973), p vi.
- ⁴¹ *Ibid.*, p v-vi.
- ⁴² Frederick Engels. *Dialectics of Nature* (Progress Publishers, Moscow, 1974), p 213.
- ⁴³ L.A. Zadeh. *Op. cit.*, p vii.
- ⁴⁴ J.W. von Goethe. *Werke* (in 14 volumes), Vol. 12 *Naturwissenschaftliche Schriften I* (Deutscher Taschenbuch Verlag, Munich, 1981), pp 172-173.
- ⁴⁵ A. Egorov. *Iskusstvo i obshchestvennaya zhizn'* (Art and Public Affairs)

(Sovetskii pisatel', Moscow, 1959), p 131.

⁸⁰ There is no unanimity on this matter in science. Apart from the hedonistic, cognitive, educational, and communicative functions of art, other functions are sometimes pointed out. E.V. Volkova, for instance, stresses satisfaction of social individuals' need for harmonisation and catharsis (see her *Proizvedenie iskusstva v sfere khudozhestvennoi kul'tury* (The Work of Art in the Sphere of Artistic Culture, Moscow, 1979, p 10). E.L. Feinberg considers that the purpose of art on the whole consists in its *discovering the force and fruitfulness of synthetic intuition, and developing a capacity for intuitive judgment as a counterweight (i.e. that offsets) the authority of the logical and in general discursive way of attaining truth*' (see his *Kibernetika, Logika, Iskustvo* (Cybernetics, Logic, and Art), Izd-vo MGU, Moscow, 1981, p 77).

⁸¹ See Nikolai Gogol. The Government Inspector, translated by Christopher English and Gordon McDougall. In N. Gogol. *A Selection* (Progress Publishers, Moscow, 1980), p 346.

⁸² Assume that an artist decides to depict in a picture the faces of males by red squares and of females by green ones, and certain natural objects by triangles of various colours and shapes, and man-made objects in the form of circles of different colours and shapes, and so on. A work of art cannot be created by such representations. It will be similar to a game of little interest in which, for example, the player who first works out the coding of the original represented will be the winner. Certain representatives of abstract art have come quite close to this.

⁸³ See entry 'Epithet' in *Bolshaya Sovetskaya Entsiklopediya*, 2nd. edition, Vol. 49 (Moscow, 1957), p 131.

⁸⁴ A. Zis. *Konfrontatsiya v estetike* (Confrontation in Aesthetics), (Iskusstvo, Moscow, 1980), p 156.

⁸⁵ André Maurois. *Prométhée ou la vie de Balzac* (Hachette, Paris, 1965), p 445.

⁸⁶ See Halldór Laxness. *Dagur i senn* (Helgafell, 1955). (Translated from the Russian in *Inostrannaya literatura*, 1957, 1: 213.)

⁸⁷ Writers employ such false identifications as a special technique for constructing a plot in a drama (Shakespeare, for example, resorted to it in *A Comedy of Errors*, *Twelfth Night*, and *The Merchant of Venice*).

⁸⁸ The list of dramatis personae in any dramatic work and the accompanying explanations and characterisations, and indications of their relations is analogous to a certain axiomatic system. In this system the names and descriptions of the characteristics of the personages ('merchant', 'Philistine', 'elderly man', etc.) may be likened to variables that get a concrete, meaningful interpretation only in the respective plays. The scheme of the relations between the characters (and, moreover, their evolution) may itself be realised in the schemes of the plots of other artistic works.

⁸⁹ In works of art like opera or ballet, the inner feelings of the characters (love, joy, ecstasy, anguish, disappointment, hatred, etc.,) are expressed not only, and not so much in their acts and behaviour as in their specific musical characteristics, and features of the production, which are aimed at evoking emotions in the addressee.

⁹⁰ See V.G. Belinsky's review of Lermontov's *A Hero of Our Time* in

- Polnoe sobranie sochinenii* (in 13 volumes), Vol. IV (Izd-vo AN SSSR, Moscow, 1954), p 203.
- ⁵⁷ See V.G. Belinsky's review of Vols. II and 12 of *Sovremennik* in *Op. cit.*, Vol. III. (Izd-vo AN SSSR, Moscow, 1953), p 53.
- ⁵⁸ *Ibid.*
- ⁵⁹ See V.G. Belinsky's article 'Shakespeare's Play *Hamlet*. Mochalov in the Role of Hamlet, in *Op. cit.*, Vol. II (Izd-vo AN SSSR, Moscow, 1953), p 290.
- ⁶⁰ A.M. Gorky. How I Learned to Write. In *Sobraniye sochinenii* (in 16 volumes), Vol. 16 (*Pravda*, Moscow, 1979), pp 285-286.
- ⁶¹ See André Maurois. *Op. cit.*, p 450.
- ⁶² V.G. Belinsky. *Art. cit.*, Vol. II, p 290.
- ⁶³ A.S. Pushkin. *O literature* (On Literature) (*Goslitizdat*, Moscow, 1962), p 446.
- ⁶⁴ See Belinsky's review of Griboedov's play *Wit Works Woe* in *Op. cit.*, Vol. III, p 436.
- ⁶⁵ *Ibid.*
- ⁶⁶ See V. Dneprov. *Problemy realizma* (Problems of Realism) (*Sovetsky pisatel'*, Leningrad, 1961), pp 3-71; A.V. Gulyga. *Iskusstvo v vek nauki* (Art in the Age of Science) (*Nauka*, Moscow, 1978), pp 17-49.
- ⁶⁷ A.V. Gulyga. *Op. cit.*, p 18.
- ⁶⁸ V. Dneprov. *Op. cit.*, p 21.
- ⁶⁹ A.V. Gulyga. *Op. cit.*, p 42.
- ⁷⁰ V. Dneprov. *Op. cit.*, pp 43-44.
- ⁷¹ A.V. Gulyga. *Op. cit.*, p 48.

CONCLUSION

ON THE SIGNIFICANCE OF GENERALISATION IN SCIENCE

There can be no knowledge or understanding without generalisation. Generalisation is a *sine qua non* of both everyday and scientific knowledge, since the latter is not satisfied by the recording of the separate and individual at the level of sensory knowledge. It is through generalisation that a person forms general concepts and general judgments, formulates norms, bans, and limitations, problems, conceptions, and theories. The generalising activity of thought lies at the basis of speech activity and is closely linked with practice, in which general rather than unique problems are usually solved. Man's thinking as a capacity to solve certain non-trivial problems is not limited to rare problems connected with satisfaction of a given subject's individual needs, but is oriented in most cases on solution of tasks of a general character that are of social significance. Discussion and verification of problems of generalisation and abstraction are important for the cybernetic modelling of cognitive processes and for advancing work on an artificial brain.

By creating broader and broader generalisations we get the chance to disclose the inner connections between the various laws already discovered by science. If we have two formulas for curves, for instance, say, $xy=1$ (of an equilateral hyperbola) and $x^2+y^2=d^2$ (of a circle with a centre at the intersection of co-ordinates), it is still not obvious from these generalisations what the relationship is in general between a circle and a hyperbola. By rising to a new level of generalisation, and elucidating a more general dependence that embraces the previous ones as partial cases, we get the chance to clarify certain new relations between the dependen-

ces mentioned.

It turns out, for instance, that the formulas cited above can be obtained from the generalised formula $ax^2 + bxy + cy^2 = d^2$ as a result of appropriate substitution of 0 and 1 for the coefficients. By resorting to ever broader generalisations we get the opportunity to formulate generalised theories, and a limited number of generalised rules for operating with the objects studied. By creating generalised theories we get the chance to explain the nature of facts that did not find explanation within the initial, narrower theory. The equality of inertial and gravitational mass, for instance, could not be explained within the context of classical physics. It was considered a chance equality, although it was a fact established empirically but not comprehended theoretically. The fact got its explanation only within a generalised theory, the theory of relativity.

Generalised theories make it possible to concretise previous, initial theories. It became clear, for example, in the light of the theory of relativity, that the laws of classical mechanics were not of universal significance; they were not fulfilled when the velocity of a moving particle approached that of light. By ascending in levels of abstraction and generalisation, we distinguish what is the most essential and important for solving some problem, and at the same time simplify it and make certain quite simple algorithms applicable to its solution.

The generalisation of certain less general concrete tasks helps to solve them more simply. This is due to the fact that good relevant generalisations are associated with discarding the inessential that obscures the essence of the matter, and with the singling out of the essential that is relevant to solution of the given problem. Generalising activity thus permeates the processes of understanding and thought, and provides science with immense advantages enabling it to reflect the world around us more profoundly and in compact form.

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